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Implications of Quality of Schooling on Economic Growth and Convergence – A System Dynamics Perspective

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Implications of Quality of Schooling on Economic Growth and Convergence – A System Dynamics Perspective

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Abstract

This paper formulates a growth model to study the interlinkages among quality of schooling, human capital and technical progress of a stylized developing economy such as India. The simulation results reveal that under the technology regimes of innovation and imitation, the quality of schooling triggers a child quantity-quality trade-off wherein parents invest in educating their children and bear lesser number of children when schooling quality exceeds an endogenously determined threshold. Consequently, the stylized economy reaches a self-sustaining growth path under both the regimes by investing in human capital of the young generation in the long-run.

Keywords: fertility; quality of schooling; economic growth; convergence; system dynamics

JEL classification: C63, J11, O11, O30.

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1 Introduction

Two of the most intriguing recent phenomena in terms of modern economic growth are the East Asian Miracle and the Latin American Puzzle. On the one hand, on average, per capita GDP grew over 4 percent in China and other East Asian countries (such as Indonesia, Thailand, Singapore, Malaysia and Taiwan) between 1960 and 1994. On the other hand, growth of per capita GDP averaged around 2 percent among other developing countries and 2.6 percent among industrialized nations during the same period (Collins, Bosworth and Rodrik, 1996). The Latin American countries registered an average per capita GDP growth rate of 1.8 percent during the period of 1960-2000. Interestingly, as Hanushek and Woessmann (2012b) discuss, Latin American region was well ahead of East Asian region in terms of the level of educational attainment (defined as average years of schooling) and income at the beginning of 1960. But surprisingly, East Asian region surpassed Latin American region in terms of economic growth by 2000. Hanushek and Woessmann (2012b) attempt to rec-oncile this puzzling finding by considering the level of educational achievement (defined as average test score on internationally comparable tests of cognitive skills) in Latin America. Their findings reveal that Latin America has done reasonably well in terms of educational attainment but it has lagged behind other regions in terms of educational achievement. The average achievement of Latin American students is lower than that of East Asian students in terms of worldwide international tests of educational achievement. This implies that, although Latin American students have been attending schools, their effective learning in schools is not high enough. The poor skill set of students in Latin America may account for one of the reasons for lack of growth of Latin American region relative to East Asian region. Similarly, Hanushek and Woessmann (2012a) have shown that quality of schooling plays a decisive role in influencing a country's economic growth.

Building upon this insight, this paper attempts to study the implications of schooling quality on economic growth of a country by integrating system dynamics numerical sim-ulation approach with the analytical approach of economics discipline. System dynamics approach has a lot of relevance in the field of economics. Most theoretical models developed in the field of economics attempt to explain the dynamic behavior of a system that is defined by inter-linkages among various endogenous variables of that system. At times, many assumptions and conditions are imposed on the analytical structure to ensure its analytical tractability. One drawback of this simplification is that it leads to a disconnect between the behavior of a system proposed by theory and the actual behavior of the system observed in real world. For instance, standard overlapping generations growth models make certain simplifying assumptions to keep the analytical structures tractable. Generally, it is assumed that individuals live for two time periods. The models assume each individual in the first age group survives to the next age group and each individual in second age group dies in the next period. The analytical structure ignores the realistic assumption of age-specific survival rates and also the fact that life spans longer than two time periods in the real world. These analytical models also assume that the price of final good is a numeraire. A SD model can tackle these limitations of the analytical models of economic growth literature. The inherent dynamic framework of system dynamics can contribute immensely in enhancing the practical working of these theoretical models.

This paper is a small step in this direction. An attempt has been made to verify and validate a growth model by running numerical simulations using the system dynamics (SD) methodology. In particular, we formulate a system dynamics model in which quality of schooling triggers a child quantity-quality trade-off at the micro level which, in turn, has repercussions on human capital formation process and technical progress at the macro level.

Unlike the analytical model, the SD model does not presupposes any long-run steady state and allows the system to reveal its out-of-steady-state behavior of variables over time. Also, the main objective of the paper is to analyze how the major macro-variables of a stylized developing economy behave over time.

This paper is organized as follows. Section 2 discusses the basic structure of the model. Section 3 discusses the model calibration and validation. Section 4 contains the key analytical results. Section 5 concludes.

2 The Model Structure

Two specific components help to describe the SD model. First, a formal algebraic description of the model is provided. Next, a causal-loop diagram has been used to explain the interrelationships and interconnectedness among major variables of the model.

2.1 *A Mathematical Description*

The SD model attempts to explain the dynamic processes of economic growth, stagnation and demographic-economic transformation of a stylized developing economy that is initiated from endogenous decisions of child bearing and education expenditure. The model structure is built upon an overlapping generations (OLG) economy, which consists of individuals who have two stages in their lives - childhood and adulthood. The population is divided into four distinct components: a) infancy to childhood (0 - 15 years); and adulthood is divided into two stages: b) young adults (15 to 44 years), c) middle aged adults (45 to 59 years) and the last stage is old age (above 60 years). The reproductive period is assumed to start at age 15 and ends at age 44. So, the young adults in the age group 15-44 take all the fertility and child bearing decisions. The middle aged people and young adults together constitute the working age population, which is actively involved in labor force participation in the production sector. It is assumed that there is no migration out of the country.

During childhood, individuals are reared and educated by their parents. All the decisions are made at the beginning of adulthood. All individuals are identical in every aspect. They inelastically supply their skills to the labor market. The individuals care about consumption, number and human capital level of their children. During old age, individuals consume their savings. The education of current period's children determines human capital endowment of next period's adult generation. It is assumed that some members leave the population with an exogenously given age-specific mortality rate given for each age group.

The model structure considers an economy consisting of two sectors. There exists the R&D sector that produces new technology using human capital as an input. The final good is produced using physical capital and technology in the final good sector. The stocks of physical capital and technology level depreciate at exogenously given rates respectively.

In light of the above discussion, for every time period, the optimization problem is formulated as follows:

$$Maximize_{c_{1t},s_t,e_t,n_t}\, u = \, log c_{1t} + \beta_1 \, log c_{2,t+1} + \beta_2 log(h_{t+1} n_t\,); \tag{1}$$

subject to

$$(w_t h_t + r_t \tfrac{K_t}{L_t})(1 - \tau n_t) = \, c_{1t} + s_t + e_t (w_t h_t + r_t \tfrac{K_t}{L_t}) n_t \tag{2}$$

$$c_{2,t+1} = \, (1 + r_{t+1}) s_t \tag{3}$$

$$h_{t+1} = (\mu + \theta e_t)^{\epsilon} \mathrm{h}_{\mathrm{t}}, \qquad \epsilon < 1 \tag{4}$$

where positive weights, β_1 and β_2, measure the importance of future consumption and child quantity and quality relative to current consumption in the utility function. τ refers to the exogenous child rearing costs in terms of a fraction of adult's time endowment, e_t is the fraction of income per child spent on education, s_t is savings and r_t is the rental rate of capital. Non-negativity constraints apply to all the variables.

The human capital of children, h_{t+1}, depends on human capital of parents, h_t, parental investment in education per child, e_t, and the quality of education system, θ, which is exogenously given. μ represents the basic skills or inter-generational human capital spillovers and ϵ measures the return to education. The first-order conditions yield:[1]

$$c_{1t} = \frac{(w_t h_t + r_t \frac{K_t}{L_t})}{1+\beta_1+\beta_2}; \tag{5}$$

$$s_t = \frac{\beta_1 (w_t h_t + r_t \frac{K_t}{L_t})}{1+\beta_1+\beta_2}; \tag{6}$$

$$e_t = \begin{cases} 0, & if\ \theta_t < \frac{\mu}{\tau\epsilon}; \\ \frac{\tau\theta - \mu}{(1-\epsilon)\theta}, & otherwise; \end{cases} \tag{7}$$

$$n_t = \frac{\beta_2 \epsilon\theta}{(1+\beta_1+\beta_2)(\mu + \theta e_t)}\ . \tag{8}$$

As shown by eqs. (7) and (8), there exists a threshold level of quality of schooling. If quality of schooling falls below the threshold, adults do not spent on child quality and maximize child quantity, n_t. This child quantity (n_t) determines the fertility rate in our stylized developing economy.

Without education expenditure, the human capital in the next generation consists of basic skills only. When the quality of schooling is high enough, such that it surpasses the threshold, an improvement in the quality of schooling triggers a child quantity-quality trade-off such that adults bear a lower number of children and invest more in the education per child in response to the improvement in the quality of schooling.[2]

The aggregate production function for final good sector in any period t is specified as:

$$Y_t = \rho A_t^{1-\alpha} K_t^{\alpha}, \quad 0 < \alpha < 1. \tag{9}$$

[1] Detailed derivation is provided in the Appendix.

[2] To see the effect when quality of schooling is above the threshold, the derivative of e_t with respect to θ in eq. (7) is given by: $\frac{\partial e_t}{\partial \theta} = \frac{\mu}{(1-\epsilon)\theta^2} > 0$.

The final homogeneous good (Y_t) is sold in a competitive market. α is the share of physical capital (K_t) in final good's production. A_t is the technology level that is produced in the R&D sector using human capital as an input. ρ is a general productivity parameter. Firms in the R&D sector employ human capital to develop new technology, which is sold at price (p_t^A). We consider two types of regimes that can drive R&D activities. The R&D sector produces new technology either by imitating from the world technology frontier or by innovating upon the local technology level. Following Papageorgiou and Perez-Sebastian (2006) and Guillo, Papageorgiou and Perez-Sebastian (2011), the production function of technology for a firm is postulated as:

$$A_{t+1} - A_t = \delta_t H_t - \eta A_t, \tag{10}$$

where $A_{t+1} - A_t$ are the new blueprints/designs produced between period t and t + 1. η is the depreciation rate of technology which is exogenously given. The productivity of R&D activity, δ_t, is constant at the firm level but at the aggregate level, it is defined as:

$$\text{Innovation regime: } \delta_t = \bar{\delta} {H_t}^{\lambda-1} {A_t}^{\phi}; \tag{11}$$

$$\text{Imitation regime: } \delta_t = \bar{\delta} {H_t}^{\lambda-1} {A_t}^{\phi} \frac{\overline{A_t}}{A_t}. \tag{12}$$

The parameter $\bar{\delta}$ denotes general productivity in R&D sector. ϕ ($0 < \phi < 1$) measures the intertemporal knowledge spillovers (standing-on-shoulders effect) and λ ($0 < \lambda < 1$) measures the diminishing returns to R&D effort (stepping-on-toes effect). $\overline{A_t}$ is the world technology frontier that grows exogenously at rate, $g_{\overline{A}}$.

$\frac{\overline{A_t}}{A_t}$ is the catch-up term which signifies the fact that greater the technological gap between leader and follower economy, higher is the potential of the follower economy to catch up through imitation of existing technologies. The returns to human capital differ between the firm (private) level and the economy-wide (social) level. There exists constant returns to R&D effort at the firm level as revealed by eq. (10). However, on the contrary, the R&D technology displays diminishing returns to R&D effort as researchers generate negative externality at the aggregate level (stepping-on-toes effect). Intertemporal knowledge spillovers (standing-on-shoulders effect) captures the positive externality of the existing R&D technology. Since all R&D firms end up in a symmetric equilibrium, the production function of technology under the innovation regime at the aggregate level reduces to:

$$A_{t+1} - A_t = \bar{\delta} {H_t}^{\lambda} {A_t}^{\phi} - \eta A_t. \tag{13}$$

Under the imitation regime, the aggregate production function simplifies to:

$$A_{t+1} - A_t = \bar{\delta} {H_t}^{\lambda} {A_t}^{\phi} \frac{\overline{A}_t}{A_t} - \eta A_t. \tag{14}$$

Firms in the R&D sector maximize their profits, given by:

$$\tau_{t,A} = p_t^A (A_{t+1} - A_t) - w_t H_t, \tag{15}$$

where p_t^A is price of a blueprint, $A_{t+1} - A_t$ are number of new blueprints discovered and w_t is the wage rate. Under both imitation and innovation regimes, using eq. (10), the profit function of the R&D firms can be expressed as:

$$\tau_{t,A} = p_t^A \left(\bar{\delta}\, H_t - \eta A_t \right) - w_t H_t, \tag{16}$$

Under both the maximization of profits leads to the following optimality condition:

$$w_t = p_t^A \delta_t. \tag{17}$$

Substituting for δ_t from eq. (13), the wage rate under the innovation regime is given by:

$$w_{IN,t} = p_t^A \bar{\delta} {H_t}^{\lambda - 1} {A_t}^{\phi} = \frac{p_t^A \, \bar{\delta} {H_t}^{\lambda} \, {A_t}^{\phi}}{H_t}. \tag{18}$$

Similarly, the wage rate under the imitation regime is given by:

$$w_{IM,t} = \frac{p_t^A \, \bar{\delta} {H_t}^{\lambda} \, {A_t}^{\phi} \frac{\overline{A}_t}{A_t}}{H_t}. \tag{19}$$

Using eqs. (13) and (14), the wage rate under both the technology regimes simplifies to

$$w_t = \frac{p_t^A \, (A_{t+1} - A_t)}{H_t}. \tag{20}$$

Since perfect competition prevails in the final good sector, the factors of production are paid according to their value of marginal product. The profit function of the final good sector can be expressed as:

$$\pi_t(Y) = p_t^Y \rho A_t^{1-\alpha} K_t^{\alpha} - r_t K_t - p_t^A \, A_t, \tag{21}$$

where p_t^Y and p_t^A are the unit prices of final good and technology respectively. The first order conditions of profit maximization yield:

$$r_t = p_t^Y \alpha \rho A_t^{1-\alpha} K_t^{\alpha - 1}; \tag{22}$$

$$p_t^A = p_t^Y (1 - \alpha) \rho A_t^{-\alpha} K_t^{\alpha}. \tag{23}$$

Inserting for Y_t from eq. (9), eqs. (22) and (23) simplify to

$$r_t = \alpha p_t^Y \frac{Y_t}{K_t}; \tag{24}$$

$$p_t^A = (1-\alpha)p_t^Y \frac{Y_t}{A_t}. \tag{25}$$

Assuming that physical capital depreciates annually at a rate denoted by ω, the next period's capital stock consists of this period's aggregate savings net of depreciation.

$$K_{t+1} - K_t = \frac{s_t L_t}{p_t^Y} - \omega K_t, \tag{26}$$

where L_t is the workforce at time period t. The goods market clearing implies that aggregate output is allocated among consumption of adults and elderly people and education expenditure and rearing costs of children in the current period and capital accumulation for the next period.

$$Y_t = \frac{c_{1t} L_t + c_{2,t-1} L_{t-1} + (\tau + e_t)\left(\mathrm{w}_t \mathrm{h}_t + \mathrm{r}_t \frac{\mathrm{K}_t}{\mathrm{L}_t}\right) n_t L_t}{p_t^Y} + K_{t+1}. \tag{27}$$

This sums up the description of the representative economy. The next section uses SD tool, that is, a causal-loop diagram (CLD) to explain the causal relationships that have been defined till now using mathematical equations.

2.2 System Dynamics Representation of the Stylized Economy

The SD model is composed of two main sub-models: the first is the demography sub-model which describes how the population in the stylized developing economy evolves over time, and the second is the economy sub-model which depicts the interrelationships between final good and R&D sectors. The entire system encapsulates the inter-relationships between these two, to capture the growth path of the economy. The aggregate system is visualized by the CLD given in Figure 1.

As shown in Figure 1, there exist five state variables in the economy, of which population and aggregate wealth are age-specific stock variables whereas per capita human capital, physical capital and technology are macro-level stock variables. Following the seminal work of Meadows, Meadows, Randers and Behrens (1972), the population is structured using an aging chain. Wealth is modeled as a co-flow of individuals who are members of the aging chain, following Torres, Lechon and Soto (2014). Fertility rate and death rate are the key variables that determine the dynamics of population growth. At the household level, quality of schooling triggers a child quantity-quality trade-off when the quality of schooling exceeds the threshold level. Here, parents invest more in the education of children and decide to have a lower number of children.

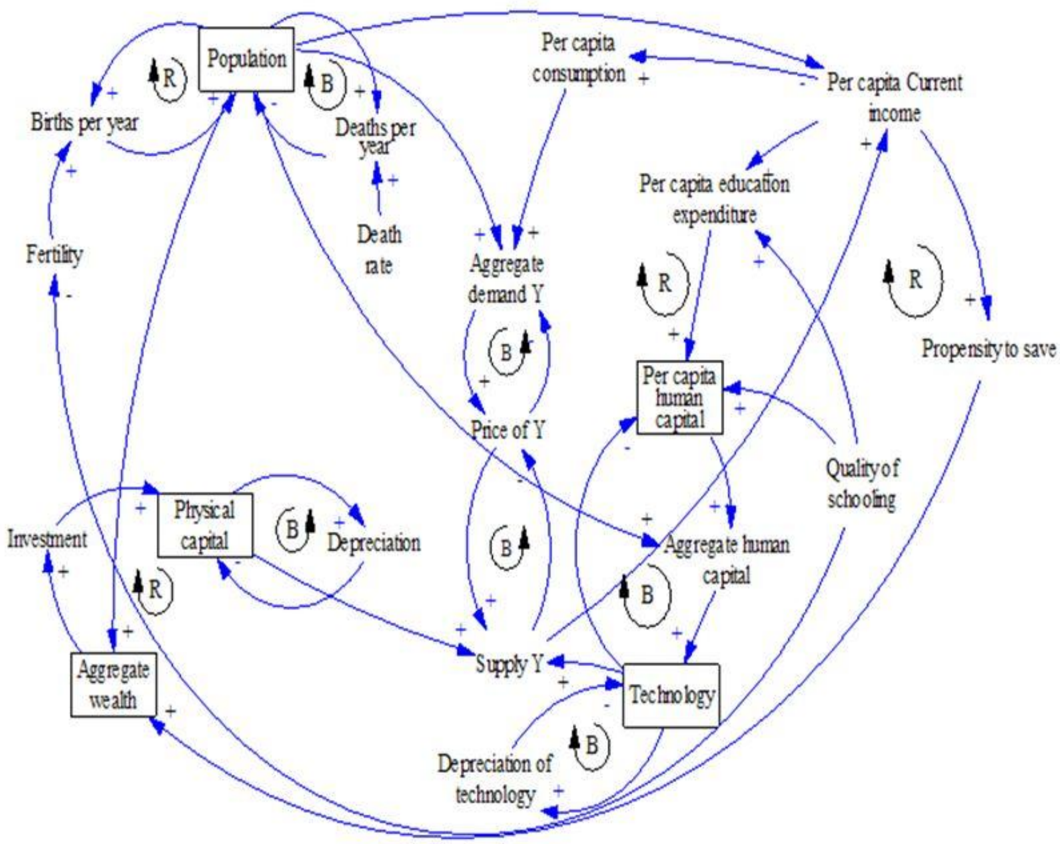


Figure 1: Causal Loop Diagram of the Stylized Economy

This leads to a decline in the fertility rate, which causes a decline in the birth rate. However, if the quality of schooling is less than the threshold level, then parents do not invest in the education of their children, and rather maximize the fertility rate, leading to a higher birth rate.

When the quality of schooling exceeds the threshold level, parents invest in the education of their children, which leads to a rise in per capita human capital, and therefore, aggregate human capital stock. Since aggregate human capital is an input for producing technology, higher stock of aggregate human capital leads to a higher stock of technology. Higher technology stock, in turn, implies higher production of output, and therefore, higher per capita income. There also exists a negative effect of rate of technical progress on human capital. A higher rate of technical progress makes knowledge obsolete, and partially erodes the positive influence of education on human capital.

Besides fertility and education investment, individuals also decide about the fraction of income that will be consumed in the current period and the amount saved for future consumption. The fraction of income saved determines the aggregate wealth of the economy. Aggregate wealth provides funds to firms for investing in physical capital. The stock of physical capital, thus, depends on the rate of investment and rate of depreciation of capital. The propensity to consume determines the aggregate demand for good Y in the current period. The price of good Y is determined endogenously to bring an equilibrium between aggregate demand and aggregate supply in the economy, that is, it is the market clearing price. Price of Y has been endogenised using Yamaguchi and Home (2014, Chapter 7) modeling approach.

This summarizes the description of our representative economy using a causal-loop structure. The results of numerical simulations are discussed in the next section.

3 Model Calibration and Validation

Model calibration is the process of setting the numerical values of parameters and initial state conditions of a model. Using India as the developing country case in point, the calibration of the model uses some data for the Indian macroeconomy. If the required data are not available for India, the parameter values are either derived from the initial state conditions or assumed based on those for similar economies. The simulation model satisfies the non-negativity constraints set in the theoretical model. However, the focus of the simulation model is not on the steady state and its comparative dynamics but instead on the transitional dynamics or out-of-steady state behavior of the variables. The model uses a year as the unit of time and takes a temporal horizon of 300 years.

The data for mortality rates have been taken from Kunte and Damani (2015). The mortality rates remain unchanged throughout the simulation. As per World Development Indicators (WDI) (2017), India's aggregate domestic savings (as percent of GDP) reached a peak of 38.3 percent in 2007 from 26 percent in 2000 and gradually declined to approximately 29 percent in 2016. Accordingly, it has been assumed that households save 30 percent of their income in each period. Accordingly, the values for various parameters, namely, preference for future consumption (β_1) and preference for child quantity and quality (β_2) are derived from the expression for savings given by eq. (6). Basic skill level, μ, which children learn through informal education from their parents is assumed to be equal to 1. Setting equal to 1 implies that children will acquire knowledge and skills atleast equivalent to their parents even when parents do not invest in the education of their children. According to the Report of United Nations, Department of Economic and Social Affairs, Population Division (2017), the total fertility rate (measured as number of children born per woman) for India has declined from 4.97 during 1975-80 to 2.3 for the period of 2015-20. Abstracting from gender differences, it is assumed that the total fertility rate per individual is 1.5. Accordingly, parametric values for quality of schooling (θ), child rearing costs (τ) and returns to schooling (ε) have been derived from eqs. (7) and (8).

The share of capital in final good sector (α) is set equal to 0.33, which is the standard assumption in growth literature (Jones, 1995; Strulik, Prettner and Prskawetz, 2013). Papageorgiou and Perez-Sebastian (2006) explain that the estimate of stepping-on-toes effect, λ, varies from 0.2 (Kortum, 1993) to 0.75 (Jones and Williams, 2000). Therefore, has been set as 0.4, which lies within the range provided by the existing literature. The standing-on-shoulders effect (ϕ) is set at 0.6 for the baseline case. Furthermore, Papageorgiou and Perez-Sebastian (2006) discuss that new technology, on an average, possesses a life-span of 10 years after factoring in the creative destruction aspect of new technology on older technology. This estimate is close to the estimate found by Caballero and Jaffe (1993). Accordingly, it is assumed that technology depreciates at the rate of 10 percent per year. Following Vandenbussche, Aghion and Meghir (2006) and Ang, Madsen and Islam (2011), US is considered as a leader economy. It is assumed that technology growth of the leader economy, $g_{\bar{A}}$, is given exogenously to the follower economy, and it is set at 2.2 percent to approximately match the average per capita growth rate of US over the post-war period. The physical capital depreciation is set at the rate of 10 percent per year. The general productivity parameter ($\bar{\delta}$) in R&D and final good sectors (ρ) are set at 0.08 and 0.5 respectively.

The next section discusses the simulation results of the baseline model.

4 Model Results

This section discusses the numerical simulation results of the baseline model for the innovation and imitation regimes of technological improvement respectively. The baseline model analyzes the temporal dynamics of a stylized developing economy in which quality of schooling exceeds the threshold. Further, taking the same baseline model, the transitional dynamics of the economy when quality of schooling is less than the threshold are also discussed as a plausible scenario in the subsequent subsection.

4.1 Evolution of Demographics of the Stylized Economy

Figure 2 represent the population size evolution over time under innovation and imitation regimes respectively. Under both the regimes, the population rises by approximately 7.7 times of the initial population by the end of the simulation period. The first cohort of children grows slowly than the other cohorts. Also, it can be seen that the middle-aged cohort grows at the lowest rate under both the regimes. Our model depicts the behaviour of a stylized developing economy. Therefore, the trend of lower population growth of the middle-aged cohort is representative of an emerging market economy, such as India, which is presently experiencing a demographic dividend. Parents invest in the education and have lower number of children as it is advantageous to educate children and augment their human capital when the quality of schooling exceeds the threshold level. At the macro level, this translates into per capita human capital growing at a higher rate as compared to the population growth rate owing to the child quantity-quality trade-off (see Figure 2). Consequently, the demography profile of the stylized economy is modified under both the imitation and innovation regimes and the working age population rises at a faster rate as compared to non-working age population[3].

This implies that the stylized economy has the potential to achieve demographic dividend under both the technology regimes as there is a shift in the age structure of population with working age population having a larger share in total population as compared to non-working age population by the end of the simulation period. This working age population with access to quality education can enhance the growth prospects of the economy.

[3] We tested for an alternative specification in which middle-aged cohort grows at a higher rate. The results do not change qualitatitively.

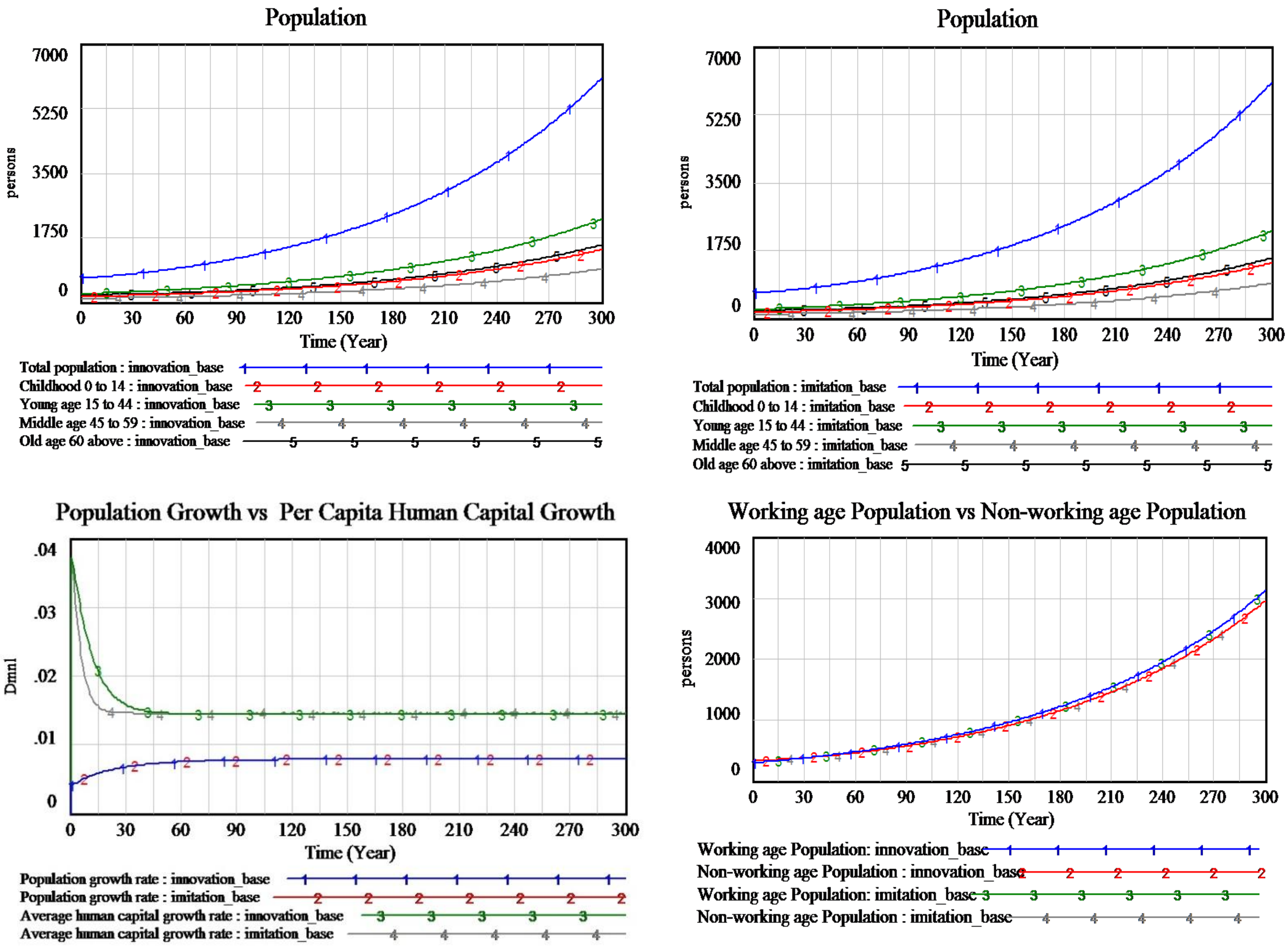
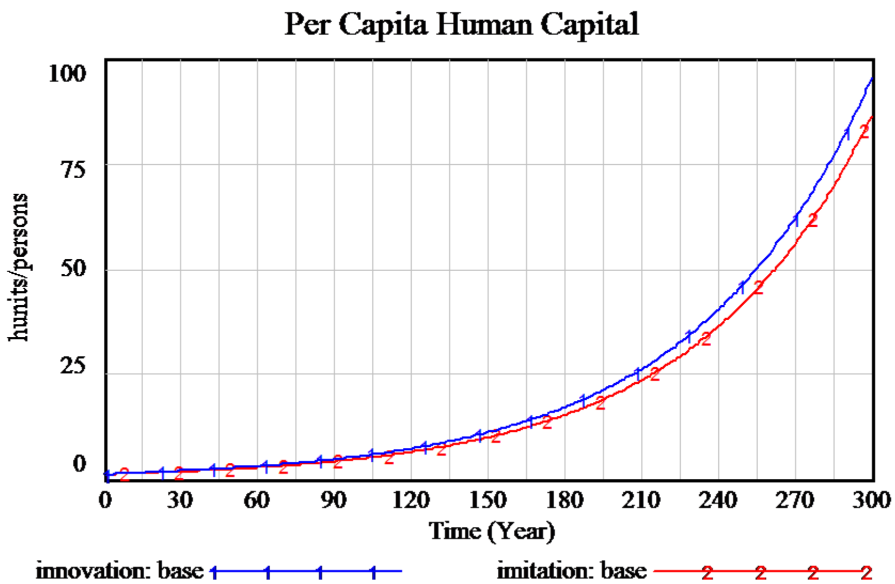


Figure 2: Demographic Structure under Innovation and Imitation Regimes

In the next section, we analyze the factors of production of our macroeconomic system.

4.2 Factors of Production of the Stylized Economy

Figure 3 displays exponential growth in per capita human capital stock over time. Since quality of schooling exceeds the critical threshold, parents educate their children, which leads to higher per capita human capital accumulation. It is notable that, although per capita human capital stock exhibits exponential trend under both the technology regimes, per capita human capital stock grows at a lower rate during the early period of simulation under imitation regime vis-á-vis innovation regime. This is on account of the “erosion effect" of technological progress on human capital. Under both the technology regimes, the same fraction of income is spent on the education of children. But, this same amount of investment augments human capital stock



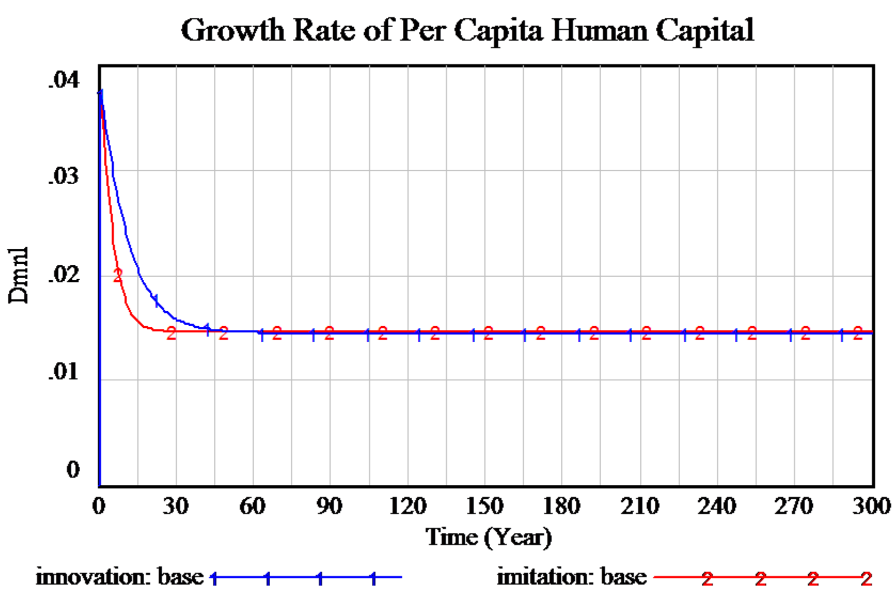


Figure 3: Per Capita Human Capital Stock under Innovation and Imitation Regimes

differently because the rate of technical progress differs initially between the two regimes, as shown in Figure 4. The rate of technical progress is higher under imitation regime vis-á-vis innovation regime during the initial period. Since a higher rate of technical progress makes knowledge obsolete and weakens the positive impact of education on human capital, the magnitude of erosion effect of technical progress is larger under the imitation regime in the initial years. Resultantly, per capita human capital level grows at a faster rate initially under innovation regime as compared to imitation regime.

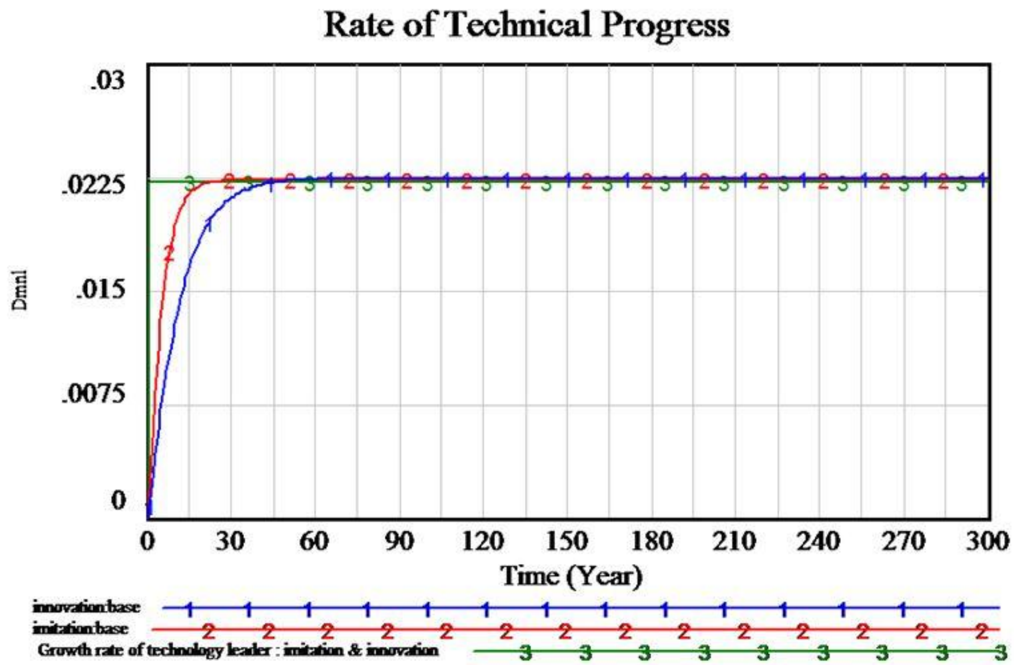


Figure 4: Rate of Technical Progress under Innovation and Imitation Regimes

Besides human capital, another factor that determines the rate of imitation is the distance of the follower economy from the world technology frontier. A technology gap between leader and follower economies implies a greater scope for imitation for the follower economy.

Therefore, due to the “catch-up effect”, an imitation economy exhibits a higher rate of technical progress in comparison to an innovation economy. As the gap closes between the imitation economy and the frontier economy, both the imitation and innovation economies grow almost at the same rate.[4]

This finding is similar to the finding of Benhabib and Spiegel (1994) and Barro (1995, Chapter 8) who show that the follower economy will grow at a higher rate than the leader until it has managed to emulate and adopt all the existing foreign technologies. As can be observed from Figure 4, the imitation economy catches up faster with the leader economy as compared to innovation economy inspite of higher erosion effect because it has been assumed that imitation is a costless activity. A larger technology gap between leader and follower economies implies

[4] However, as will be shown in the subsection on sensitivity analysis, the rate of technical progress exhibited under the two technology regimes is dependent on the level of parameters chosen.

larger "catch-up effect" which outweighs the erosion effect of technical progress on human capital. As a result, the imitation economy is able to catch up faster with the frontier economy.

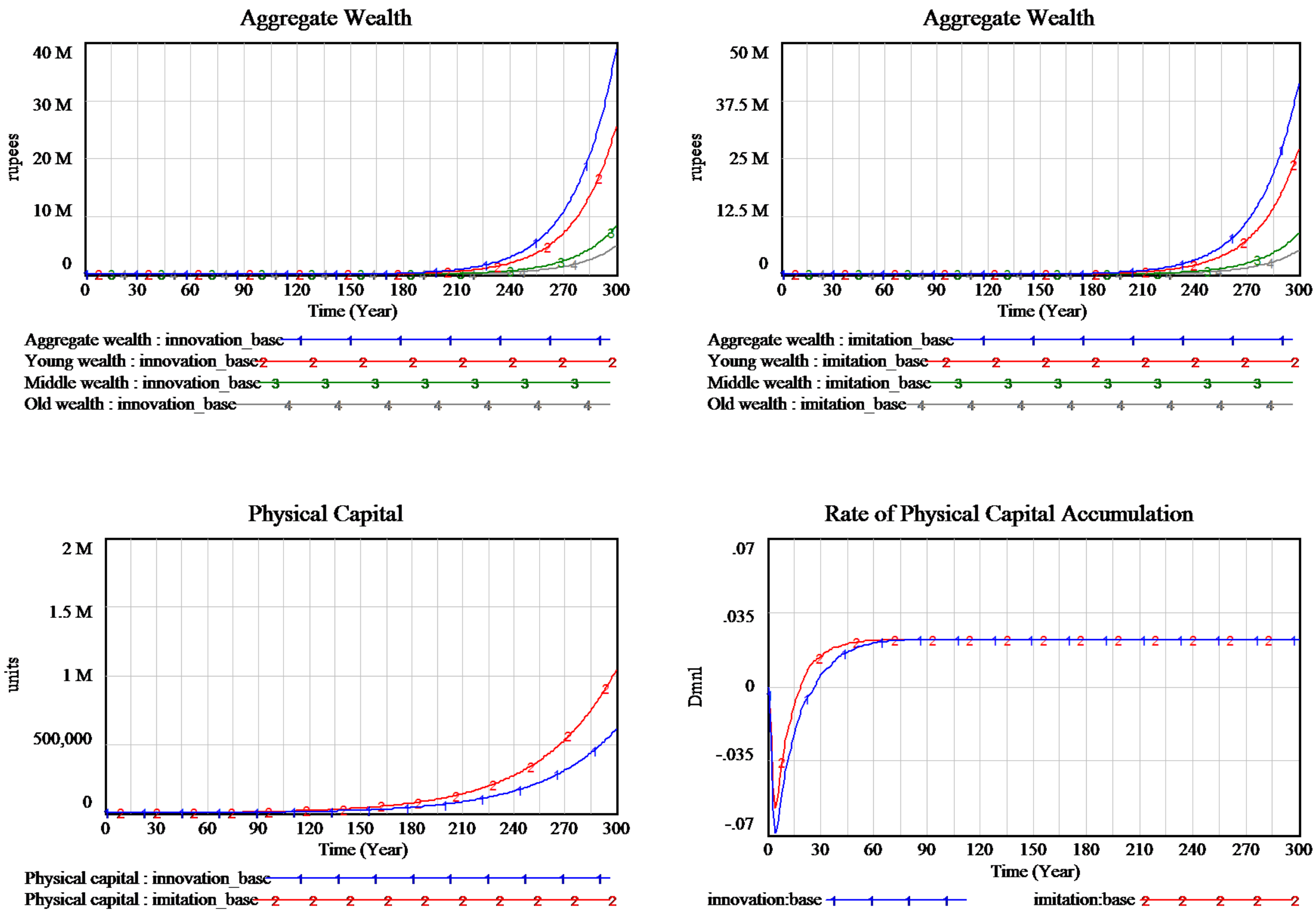


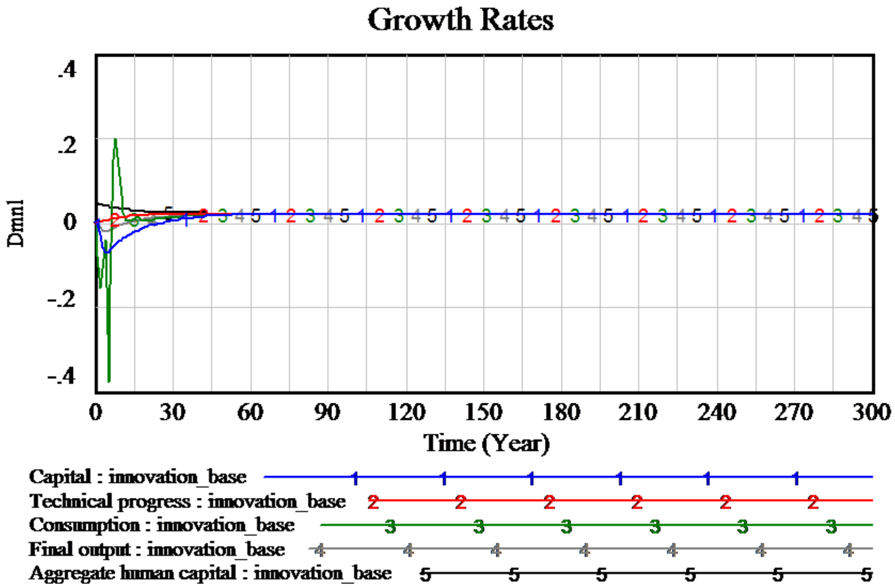
Figure 5: Aggregate Wealth and Total Physical Capital under Innovation and Imitation Regimes

We next analyze the growth pattern of physical capital of our model economy. Physical capital accumulates when a higher level investment is undertaken in physical capital over time. Households provide funds to firms for investing in physical capital. As displayed in the Figure 5, the total stock of wealth grows over time because per capita savings increase as per capita income rises over time. The wealth of young people grows faster than that of middle aged people. Since elderly consume off their savings, their wealth increases gradually. As Ghani (2011) point out, this is another channel through which demographic dividend can in influence growth potential of an economy as higher savings facilitate accumulation of physical capital and technological innovation. Since, it is the young and middle aged people who lend their money to firms for investment , physical capital stock depicts an exponential trend akin to aggregate wealth under both the technology regimes. We next characterize the steady state of the economy under the two technology regimes.

4.3 Growth Rates along the Steady State

Figure 6 displays the steady state growth rates of major variables of the economy under these regimes. As can be observed, all the macro variables grow at the same rate under innovation regime. Similarly, physical capital, aggregate output, aggregate consumption grow at the same rate as the rate of technical progress under imitation regime. Technology grows at a higher rate initially under the imitation regime due to the "catch-up effect". Under both the regimes, the self-sustaining growth path of the stylized economy in the long-run is driven by the rate of human capital accumulation when quality of schooling is high enough to surpass the threshold value. Intuitively, when quality of schooling surpasses the threshold, it has two opposing effects on human capital accumulation. First, parents invest in education of children which stimulates the accumulation of human capital which fosters technical progress leading to a higher economic growth in the economy. This is the growth-stimulating effect. Second, the increase in education is also accompanied by a decline in fertility rate. This constitutes the growth-impeding effect that reduces the total factor productivity growth and economic growth by contracting the pool of available researchers. The growth-stimulating effect overpowers the growth-impeding effect of quality of schooling when quality of schooling is high enough to surpass the threshold value. Therefore, quality of schooling fosters human capital accumulation and raises total factor productivity growth, putting the economy on to a self-sustaining growth path in the long-run.

An economy with a better-quality human capital stock is more resilient to technical change. As per the Draft of World Development Report (2019), technology is changing the skills required at work in the present world. Since 2001, the share of employment in cognitive skills-intensive jobs has increased from 19 to 23 percent in developing countries and from 33 to 41 percent in developed countries. On the other hand, the employment share in jobs intensive in routine skills has declined from 50 to 44 percent in developing countries and from 42 to 32 percent in developed countries. This rapidly evolving technology landscape has generated greater demand for high-skilled workers which, in turn, can be met only by imparting quality education to the young generation.



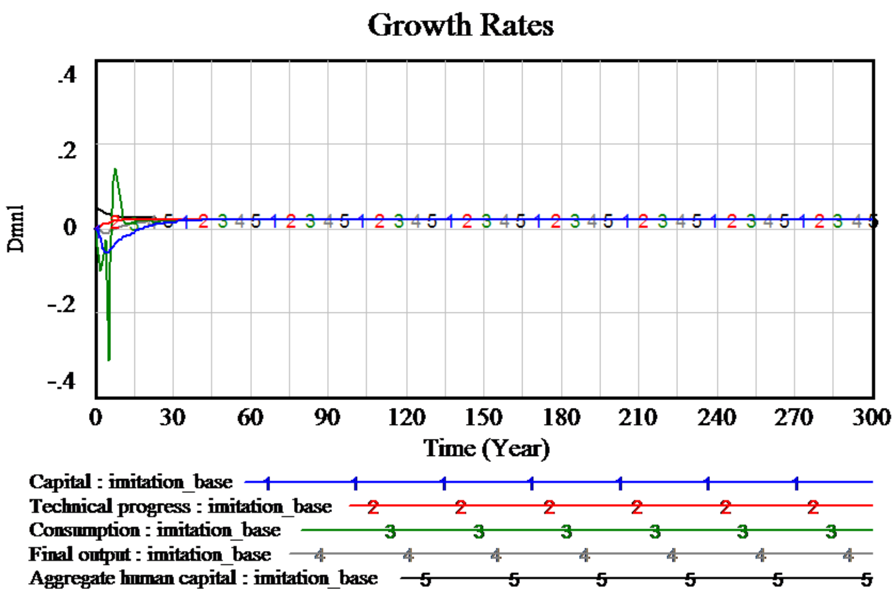


Figure 6: Growth Rates of Major Macro Variables under Innovation and Imitation Regimes

Another advantage of better-quality human capital stock is that it can enable a developing economy to tap its demographic dividend. Demographic dividend without investments in education is a lost opportunity as the growth benefits stemming from a demographic dividend will fructify only when the working-age population acquires the requisite skills through quality education and seeks gainful employment. In this scenario only, the stock of human capital can generate higher economic growth. Therefore, quality of education has far-reaching implications on the growth prospects of an economy.

After examining the results of the baseline model, it can be concluded that the system dynamics model does not show any behavioral inconsistencies. Under both the regimes, the model economy attains the steady state after a period in time.

The next subsection discusses the results of sensitivity analysis that has been conducted to check the robustness of the results of the baseline model.

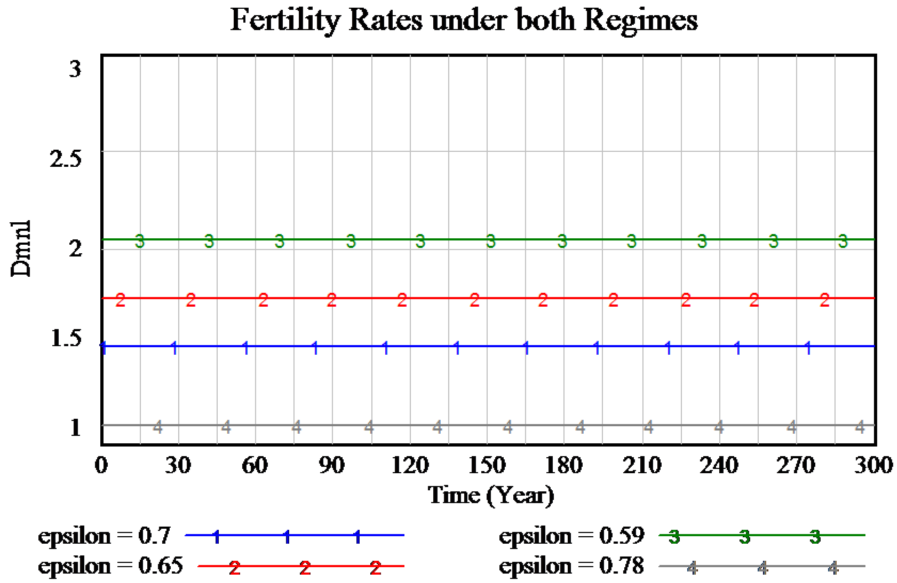
4.4 Sensitivity Analysis of the Baseline Model

4.4.1 Sensitivity Analysis w.r.t Returns to Education (ε)

From Figure 7a and 7b, it is straightforward to observe that, an increase in returns to education yield higher rate of technical progress, and therefore, economic growth under both innovation and imitation regimes. Intuitively, the threshold value of quality of schooling , $\frac{\mu}{\tau\epsilon}$ is decreasing in the value of ε. This implies that, ceteris paribus, the critical threshold value of quality of schooling decreases as returns to schooling increase when quality of schooling exceeds the threshold (see Figure 7a). Therefore, parents educate their children and bear lower number of children in response to an increase in returns to education.

At the macro level, this micro level trade-off generates a growth-stimulating effect and a growth-impeding effect. Investment in education of children stimulates the accumulation of human capital which fosters technical progress leading to higher economic growth. This is the growth-stimulating effect. The increase in education is also accompanied by a decline in fertility rate as returns to education increase. This constitutes the growth-impeding effect that reduces the total factor productivity growth and economic growth by contracting the pool of available researchers. The growth-stimulating effect overpowers the growth-impeding effect of a change in returns to education when quality of schooling exceeds the threshold. Resultantly, an increase in returns to education yield higher rate of technical progress and therefore, economic growth under both innovation and imitation regimes of technological improvement.

Further, it can be observed from Figure 7b that when returns to schooling assume a high enough value (say, 0.78 from 0.7), the imitation economy transitions to an innovation regime after approximately 50 years. When ε assumes a higher value, per capita human capital rises at a faster rate. This higher rate of human capital accumulation makes it possible for the imitation economy to close the gap with the world technology frontier more quickly as parents spend a higher fraction of income on education of a child due to higher returns to education. Once the



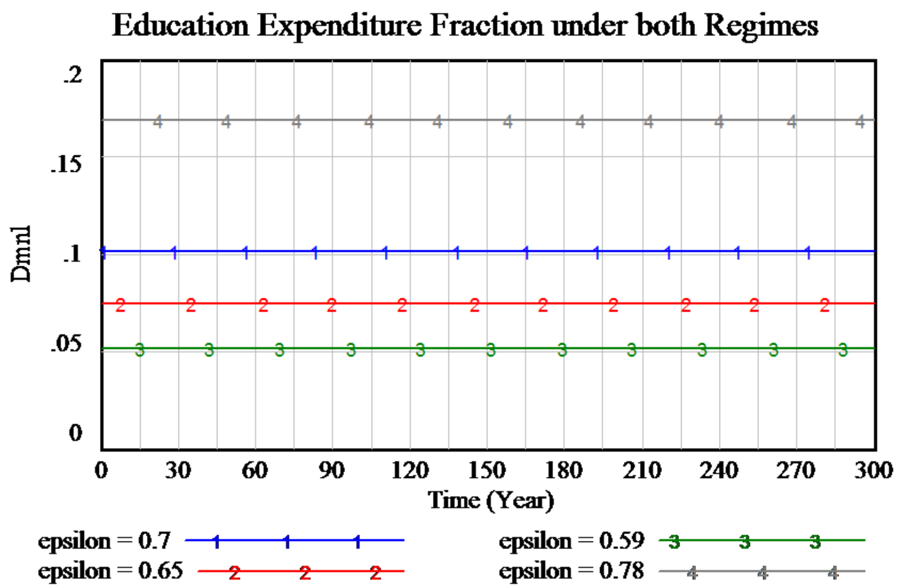


Figure 7a: Sensitivity Analysis w.r.t Returns to Education under Innovation and Imitation Regimes

imitation economy bridges the technology gap with the world frontier, it starts innovating on the local technology frontier. Resultantly, there is a jump in the rate of technical progress for the imitation economy after approximately 50 years as this economy transitions to innovation economy. A simultaneous decline in growth rate of average human capital is observed under imitation regime after 50 years owing to erosion effect of technical progress. As explained earlier, a higher rate of technical progress implies a higher erosion effect on per capita human capital. After catching up with the world frontier, the imitation economy is now the leader economy, and it exhibits a higher rate of technical progress as it innovates on the local frontier. As a consequence, per capita human capital growth rate falls due to this higher erosion effect.

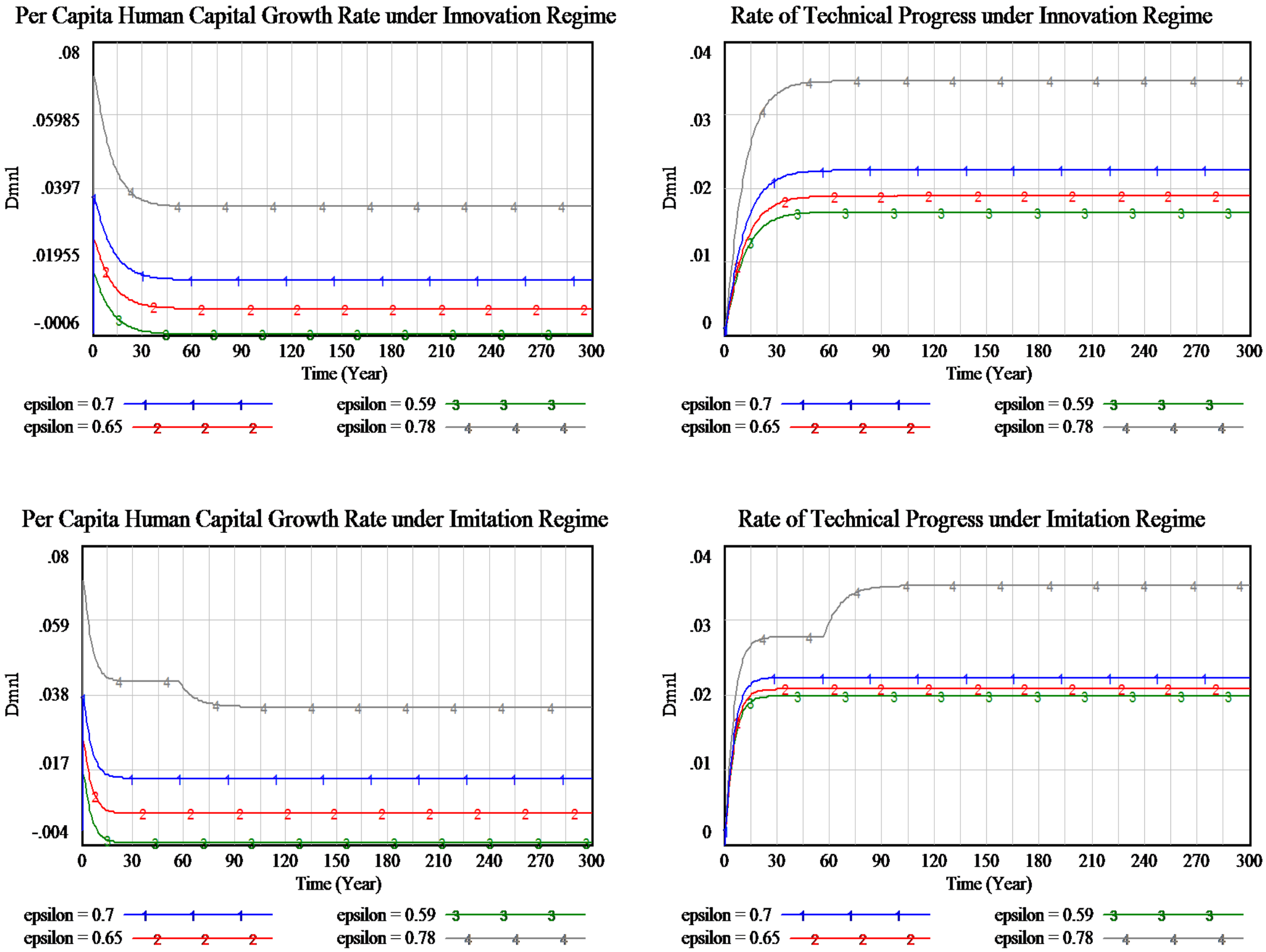


Figure 7b: Sensitivity Analysis w.r.t Returns to Education under Innovation and Imitation Regimes

Another notable point is that the growth rate of human capital becomes negative when returns to schooling are low. This is due to the fact that parents spend a very small fraction of income on the education of their children in this particular case. This negligible investment in education is not enough to offset the erosion effect of technical progress on per capita human capital. As a result, growth rate of per capita human capital becomes negative under both the technology regimes. However, negative growth rate of human capital is higher under imitation regime as compared to innovation regime because imitation economy exhibits a relatively higher rate of technical progress via the “catch-up effect". Resultantly, there is a higher erosion effect under imitation regime, which leads to a higher negative growth of per capita human capital in the imitation economy.

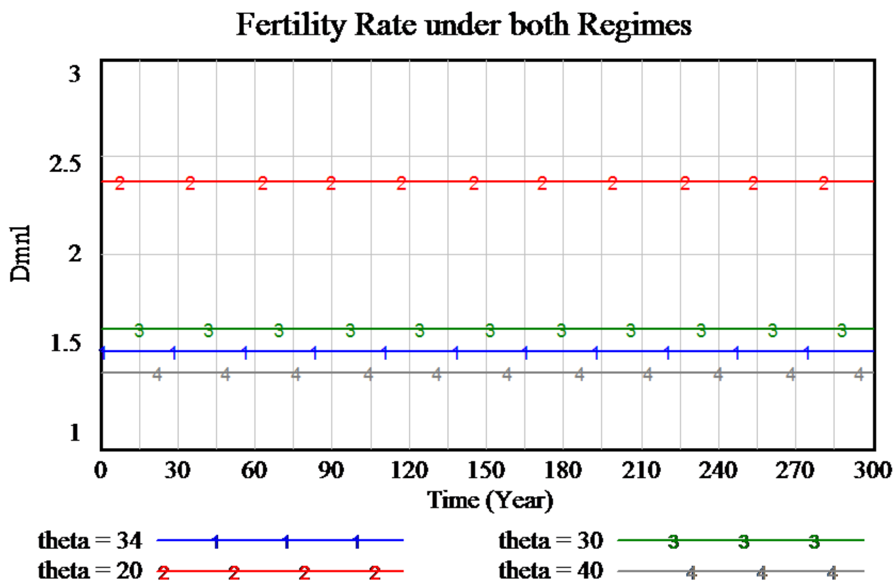
Using Mincerian approach, Psacharopoulos and Patrinos (2018) provide estimates for private rate of return to an additional year of schooling for different countries during the period of 1950-2014. The average private return to schooling for the world as a whole is estimated to be around 8.8 percent. Furthermore, the private returns to schooling are higher in low-income countries relative to high-income countries reflecting the scarcity of human capital in the low-income countries. In particular, return to schooling is found to be 9.3 percent with a mean years of schooling of 5 years for low-income countries. The corresponding estimates of returns to schooling for middle-income and high-income countries are 9.2 and 8.2 percent with means years of schooling of 7 and 9.2 years respectively.

However, it should be noted that these empirical estimates are for marginal returns to schooling. In our model, ε does not represent marginal returns to schooling but instead, it is one of the parameters that determines marginal returns to schooling along with a bunch of other parameters. . As explained in Section 3, the parametric value of returns to schooling (ε) has been calculated through the process of calibration in order to get a fertility rate of 1.5 per individual abstracting from gender differences. The objective is to show the long-run dynamics of a stylized developing economy that has a fertility rate closer to the replacement level of fertility.

We, next, discuss the results of sensitivity analysis with respect to quality of schooling.

4.4.2 Sensitivity Analysis w.r.t Quality of Schooling (θ)

As can be seen in Figure 8a, an increase in quality of schooling leads to a decline in fertility rate and increase in fraction of income spent on education under both the technology regimes. Accordingly, an increase in growth rate of per capita human capital and growth rate of technology is observed for both types of economies as quality of schooling increases (see Figure 8b). When quality of schooling assumes a high enough value, it can be seen that the imitation economy is able to bridge the technology gap with the leader economy and transitions to the innovation regime leading to a jump in the rate of technical progress. There is a simultaneous decline in the growth rate of per capita human capital due to a higher erosion effect of technical progress after an imitation economy switches to innovation. This occurs because of the same reason explained for the case of high returns to education in the previous subsection. Similar to the effect of lower returns to schooling, it can be seen that the growth rate of human capital becomes negative when quality of schooling is low.



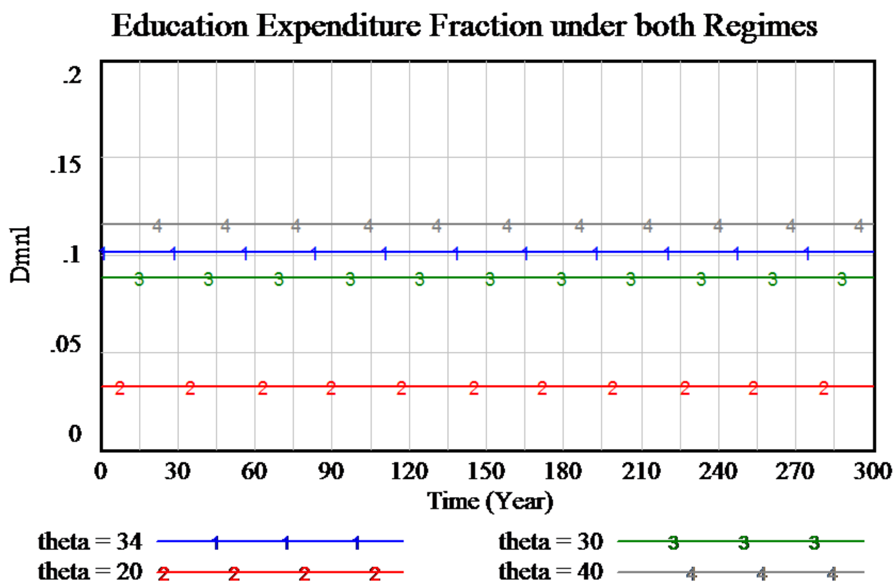


Figure 8a: Sensitivity Analysis w.r.t Quality of Schooling under Innovation and Imitation Regimes

Intuitively, similar to returns to education, an improvement in quality of schooling generates a growth-stimulating and a growth-impeding effect at the macro-level. Total factor productivity growth and economic growth will accelerate or decelerate depending upon the relative magnitude of the two effects. The growth-stimulating effect overpowers the growth-impeding effect of a change in quality of schooling when it exceeds the threshold. Therefore, rate of technical progress rises as quality of schooling improves. However, technical progress makes knowledge obsolete and has a dampening effect on per capita human capital. Therefore, quality of schooling needs to be high enough to ensure that adequate investment is done in human capital such that the growth-stimulating effect of schooling quality overpowers both the growth-impeding effect of schooling quality and the erosion effect of technical progress to warrant a positive rate of human capital accumulation and therefore, higher economic growth. A mere surpassing of threshold level of quality of schooling cannot ensure a high economic growth rate for the two types of economies[5].

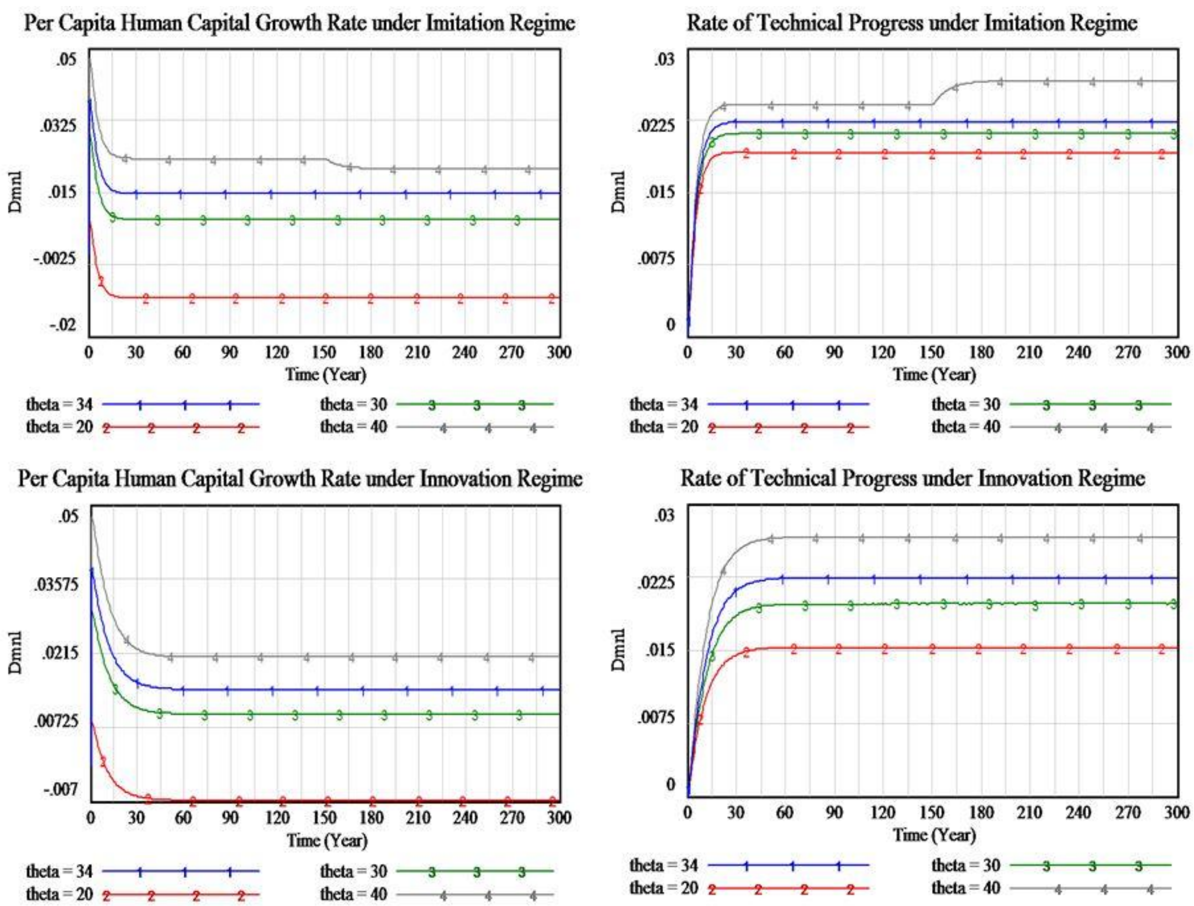


Figure 8b: Sensitivity Analysis w.r.t Quality of Schooling under Innovation and Imitation Regimes

[5] Quality of schooling is exogenous in our present model. However, we have developed another model (which is the subject matter of our other paper) in which quality of schooling has been endogenized. In that extended model, the quality of schooling is defined as a by-product of the twin processes of physical capital accumulation and per capita human capital formation in the economy, even as government sector is not modelled explicitly. We believe that modeling schooling quality as a function of physical capital and per capita human capital is akin to public investment on education for improving schooling quality. Our simulation results show that the stylized economy attains self-sustaining growth path fuelled by human capital accumulation.

This is somewhat similar to Galor and Moav (2000) who analyze this in a theoretical context. They postulate that able individuals have a comparative advantage in adapting to new technologies. The time required to learn new technologies diminishes with the level of ability. Similarly, Galor (2005) theorizes that human capital accumulation and technological progress are positively correlated as higher level of schooling mitigates the adverse effect of technological progress on human capital accumulation.

4.4.3 Sensitivity Analysis w.r.t Technological Growth Rate of Leader Economy ($g_{\bar{A}}$)

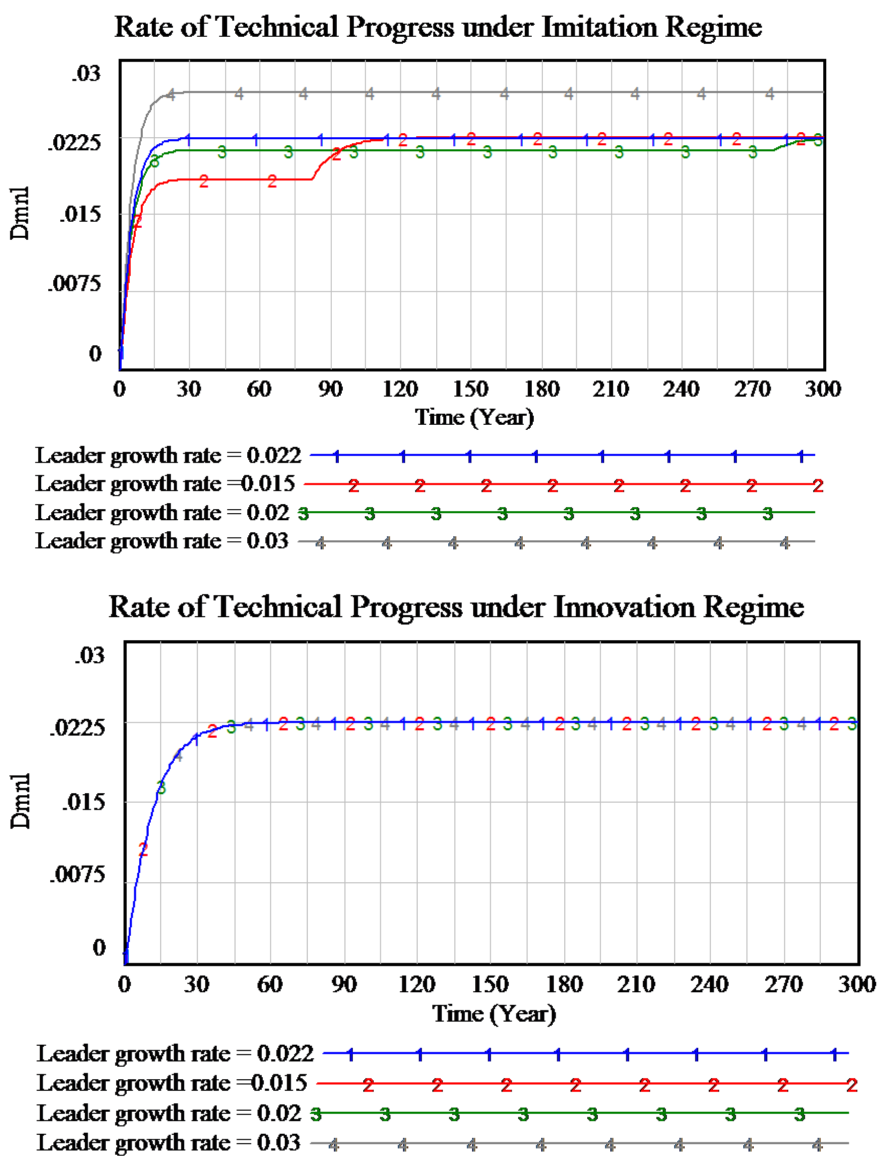


Figure 9: Sensitivity Analysis w.r.t Technological Growth Rate of Leader Economy under Innovation and Imitation Regimes

Figure 9 shows the sensitivity results for both types of economies. On the one hand, under innovation regime, there is no effect of change in growth rate of leader economy on rate of technical progress as innovation economy innovates upon its own local technology frontier, which has no relationship with the technology frontier of the world. On the other hand, the rate of technological improvement under imitation regime is directly related to growth rate of technology frontier. If the technology frontier grows faster, then there is greater scope for imitation and, therefore, imitation economy grows at a faster rate. However, it is still not able to converge to the global frontier because convergence to a rapidly growing global frontier requires a higher rate of human capital accumulation, which is not ensured by the existing level of quality of schooling. Quality of schooling needs to be higher than the existing level so that parents expend higher fraction of income on education of their offspring that would, in turn, ensure a higher rate of human capital accumulation. Alternatively, if the world technology frontier grows at a smaller pace, then imitation economy grows at a faster rate and surpasses the world frontier. Once it surpasses the world frontier, the technology gap between world frontier and follower is closed and it starts innovating on local technology frontier. Thus, time taken to catch up with the frontier depends on the rate at which frontier is growing. Therefore, it can be concluded that the imitation economy may or may not converge to the world frontier depending on the rate of growth of world technology frontier.

4.4.4 Sensitivity Analysis w.r.t Inter-Generational Human Capital Spillovers /Basic Skills (μ)

It can be seen from Figure 10b that an increase in inter-generational human capital spillover (basic skills) leads to lower rate of technical progress and, therefore, lower economic growth under both the regimes of technological improvement. Intuitively, the threshold value of quality of schooling , $\frac{\mu}{\tau\epsilon}$ is increasing in the value of μ. This implies that, ceteris paribus, the critical threshold value of quality of schooling increases as inter-generational human capital spillovers (basic skills) increase when quality of schooling exceeds the threshold. Therefore, parents make lower investment in the education of their children and, instead, choose to bear higher number of children in response to an increase in inter-generational human capital spillovers (see Figure 10a). At the macro level, a lower investment in education of children leads to lower rate of accumulation of human capital leading to lower rate of technical progress and economic growth under both the innovation and imitation regimes.

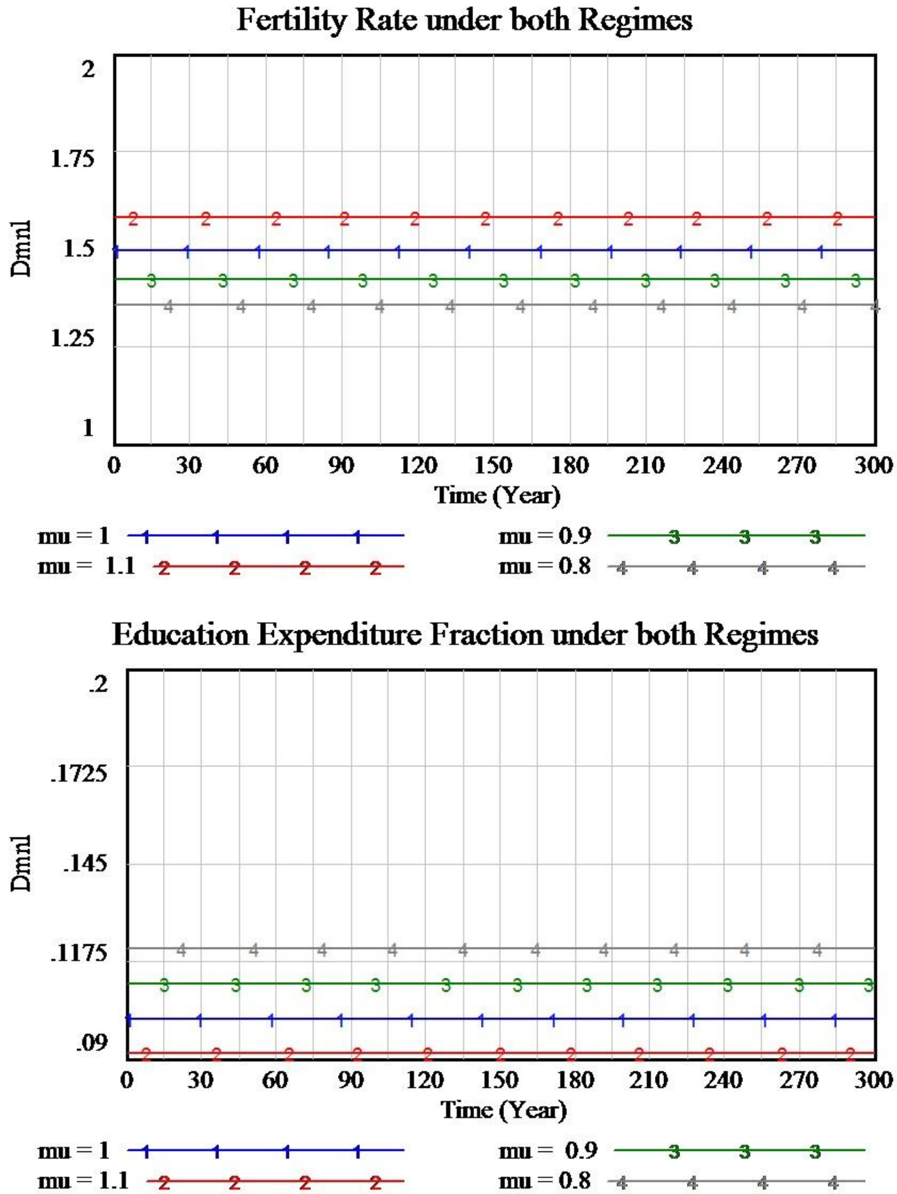


Figure 10a: Sensitivity Analysis w.r.t Inter-Generational Human Capital Spillovers under Innovation and Imitation Regimes

Inter-generational human capital spillovers are basically skills learnt by children by observing and imitating parents. These imply that children will acquire knowledge and skills atleast equivalent to their parents when parents choose not educate their children in an outside institution. A high enough value of inter-generational human capital spillovers implies that children acquire/ inherit a lot of knowledge and skills from their parents. In this case, parents can be induced to invest in the education of their offspring only if the quality of schooling is considerably high. The quality of schooling is exogenous in our model. Therefore, ceteris paribus, an increase in inter-generational spillover leads to lower investment in education and, hence, a lower rate of technical progress and economic growth under the two technology regimes in this stylized model.[6]

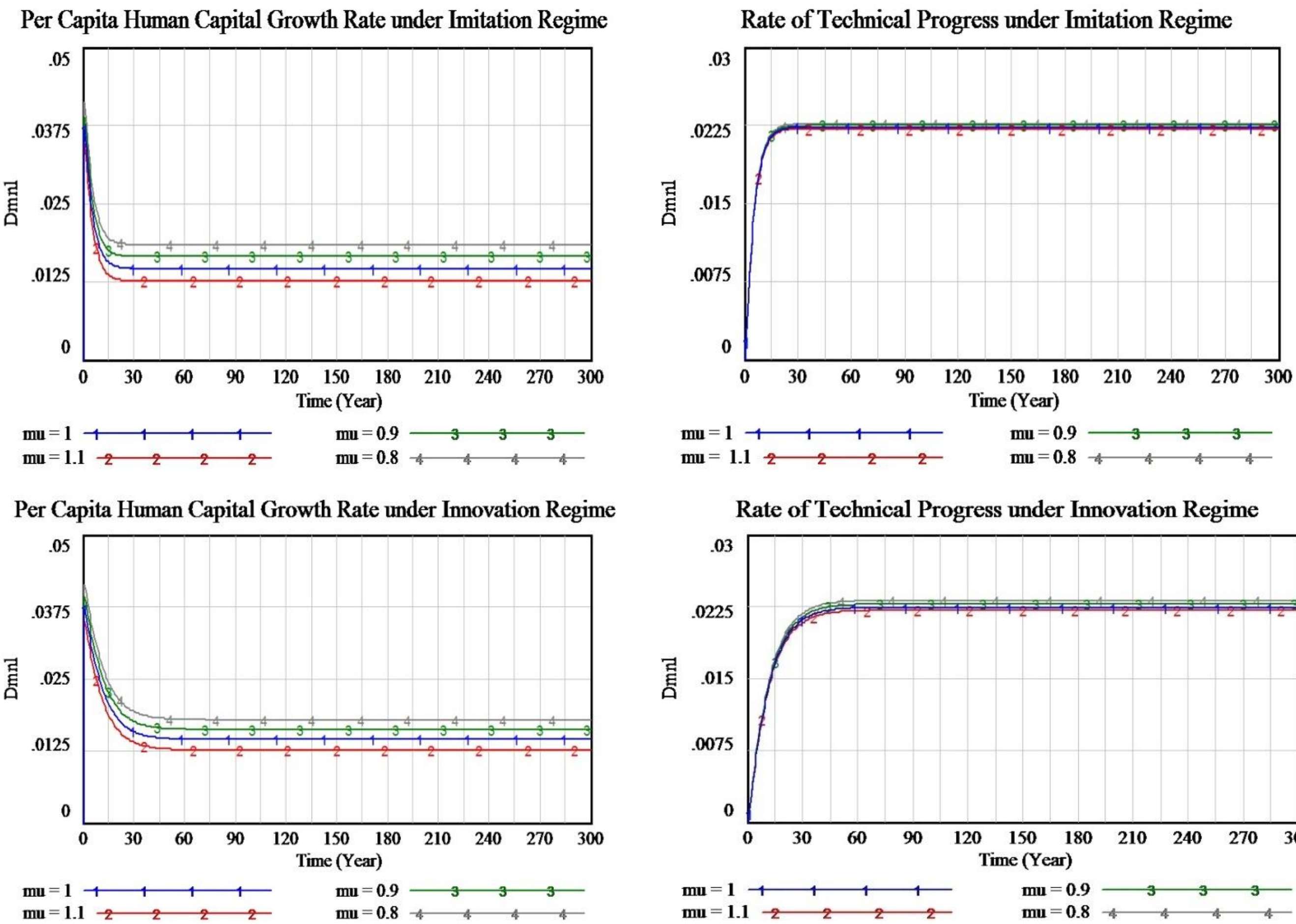


Figure 10b: Sensitivity Analysis w.r.t Inter-Generational Human Capital Spillovers under Innovation and Imitation Regimes

Furthermore, it can be observed from Figure 10b that the rate of technical progress under imitation regime is relatively less sensitive to changes in inter-generational human capital spillovers as compared to innovation regime because the growth rate of technology frontier, which is another factor that determines technical progress in imitation economy, partially offsets the impact of change in human capital spillovers on the rate of technical progress under imitation

[6] We gratefully acknowledge an anonymous referee for pointing out the implications of inter-generational human capital spillover.

regime. As a result, growth rate of technology under imitation regime is relatively less affected by changes in human capital spillovers.

In light of these sensitivity results, it can be concluded that the rate of technical progress under both the technology regimes depends on various parameters which are specific to that particular economy. This implies that the convergence of the follower economy under both imitation and innovation regimes is contingent upon the economy-specific characteristics. Higher quality of schooling and returns to education facilitate the process of convergence to the world technology frontier whereas the time taken to catch up with the global frontier depends on the rate at which this frontier is growing. A country with higher quality of schooling has a higher probability of converging to the world frontier as compared to an economy with lower quality of schooling. On the one hand, if the world frontier is growing at a fast enough pace, then the possibility of convergence is low. On the other hand, the follower economy can converge to the world frontier if the world frontier expands at a low enough rate. This result is in line with Basu and Mehra (2014).

Taking this baseline model as a reference point, the next section discusses an alternative scenario for both imitation and innovation regimes.

4.5 An Alternative Scenario : Quality of Schooling is less than the Threshold ($\theta < \mu/\tau\varepsilon$)

An alternative scenario of quality of schooling lower than the threshold is used to further explore the dynamics of our model. In this case, it is assumed that the numerical values of all the parameters remain the same as in the baseline model except the quality of schooling, which now assumes a value less than the critical threshold ($\frac{\mu}{\tau\epsilon}$). The impact of quality of schooling being less than the critical threshold value on population and average human capital can be clearly seen from Figure 11.

As can be seen, in comparison with the baseline scenario, population now grows at a higher rate under both the regimes. When quality of schooling is less than the threshold, parents do not educate their children and focus on maximizing fertility instead. As is shown in Figure 11, fertility rates are higher and a zero fraction of income is allocated towards education of children under both the technology regimes. Accordingly, per capita human capital stock falls under both the regimes as children only acquire basic skills and knowledge from their parents in the absence of investment in education. Also, per capita human capital level degrades over time due to

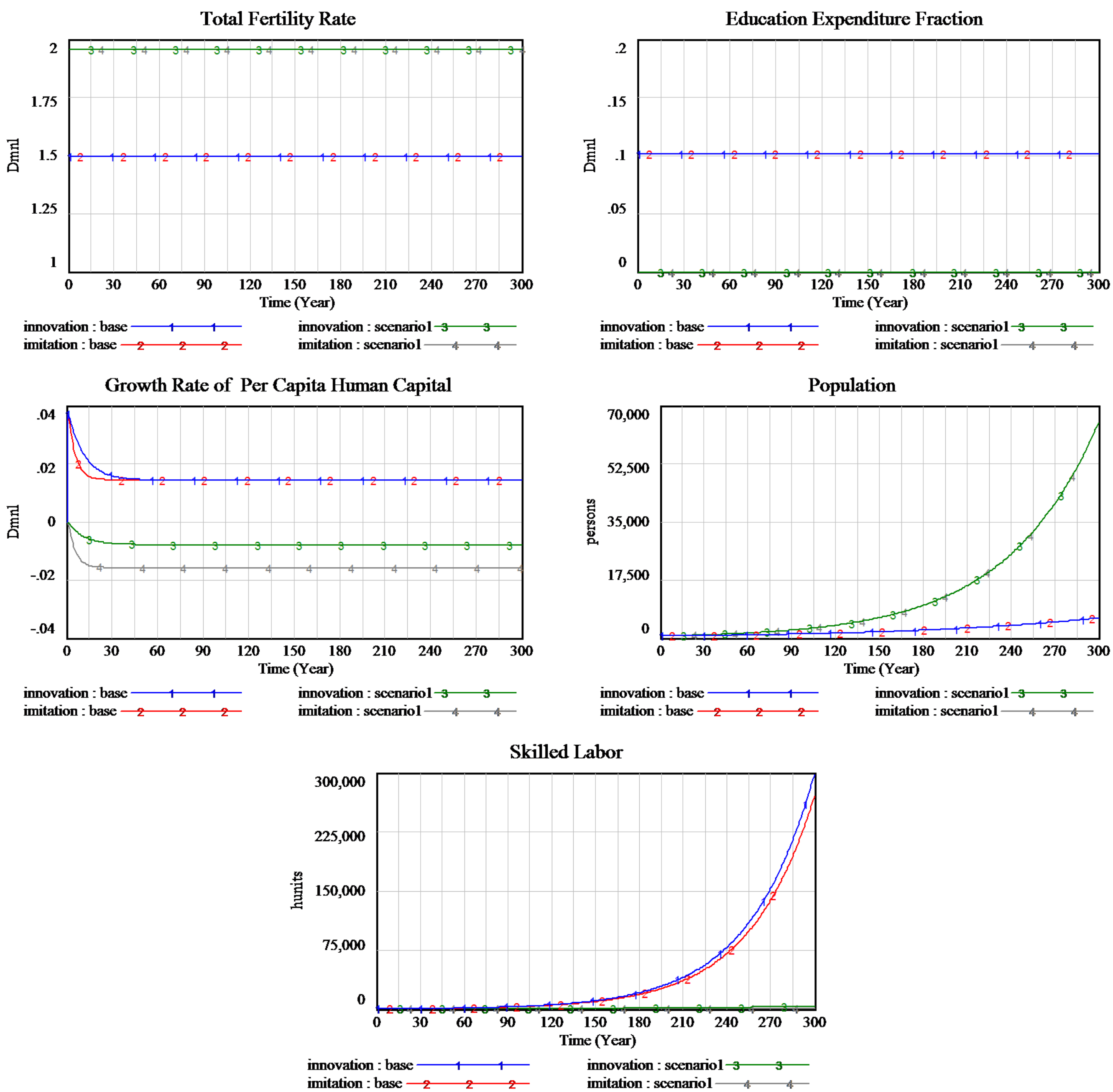


Figure 11: An Alternative Scenario: Quality of Schooling is less than the Threshold

erosion effect of technical progress. The skilled labor grows marginally under both the technology regimes in this alternative scenario. However, it should be observed that, initially, the decline in per capita human capital level is greater for imitation economy vis-á-vis innovation economy. This occurs because now a very technologically backward economy is being considered where there is no investment in human capital. Under imitation regime, there is a large scope for imitating the technology via “catch-up" effect for this backward economy.

Therefore, due to the “catch-up" effect, imitation economy exhibits a higher rate of technical progress in comparison with innovation economy, as visualized in Figure 12. Consequently, stock of per capita human capital declines relatively more due to larger erosion effect of technology on human capital under imitation regime initially.

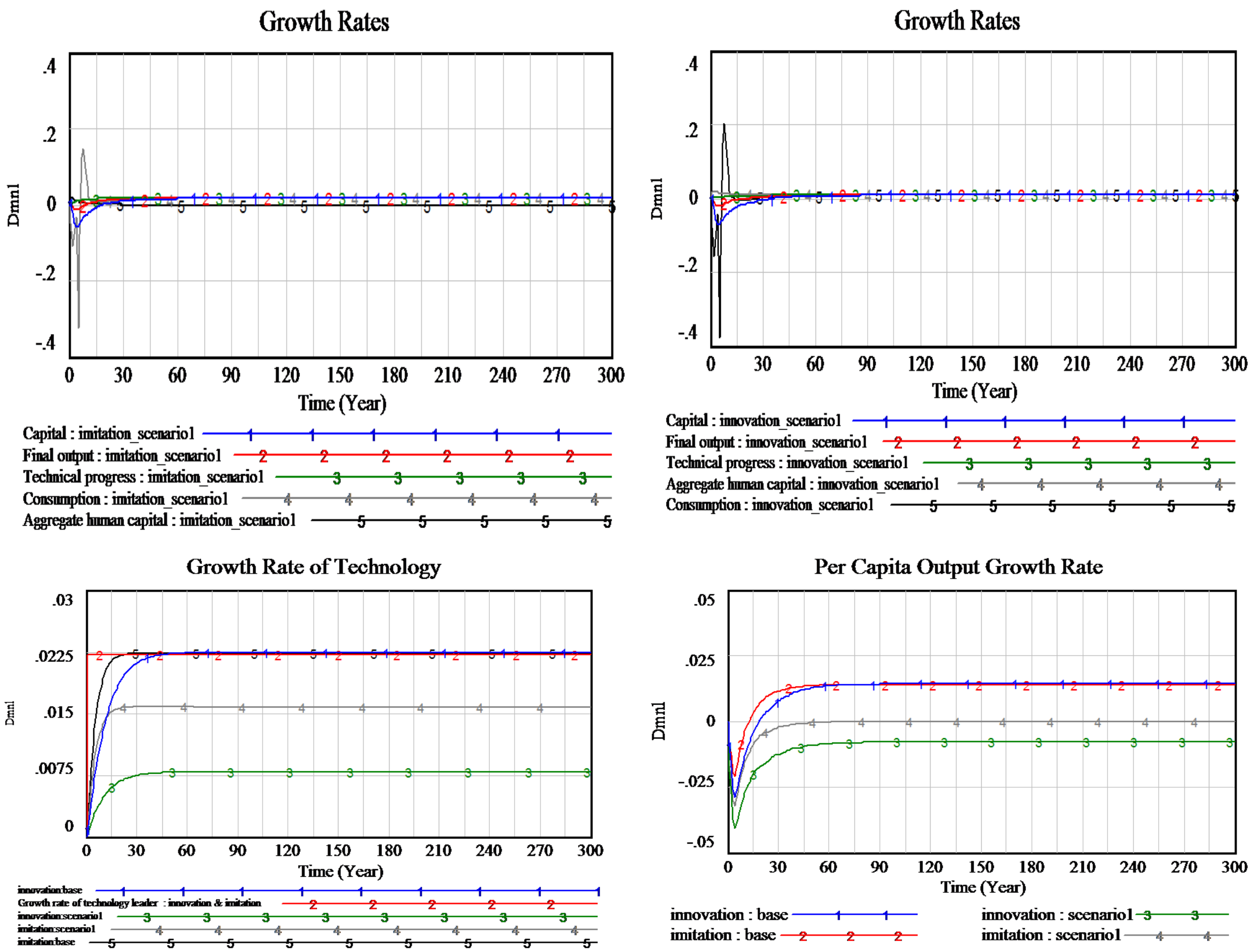


Figure 12: An Alternative Scenario: Quality of Schooling is less than the Threshold

As depicted in Figure 12, both types of economies are not able to converge to the frontier due to the abundance of low-skilled workers, who do not possess the skills needed in R&D sector for innovating or imitating technologies. Workers are not able to acquire the needed skills through formal schooling as parents do not educate their children due to low quality of schooling. This result is in line with the empirical findings of Hanushek and Woessmann (2012a). They look at the distribution of scores by defining two variables that measure the proportion of students that meet a threshold level of achievement. The first is a score of 400 or above on the transformed international scale, that is, one standard deviation below the mean test scores for OECD countries (meant to capture basic literacy) and the other 600 or above (to capture high achievement). They find that the effect of basic-literacy share does not vary significantly with the initial level of development, but the effect of high achieving share of students is significantly larger in countries that have more scope to catch up to the most technologically advanced countries. From this perspective, it can be inferred that the process of economic convergence is accelerated in countries with larger shares of high performing students. Therefore, countries need human capital with high cognitive skills to improve their prospects for convergence to the world technology frontier. In a similar context, Stokey (2015) show that a stylized economy exhibits sustained growth in the long-run if the barriers to technology inflows are low and government subsidies to promote human capital accumulation are high. On the contrary, if the technology barriers are sufficiently high and human capital promoting subsidies are low, then the economy stagnates and converges to minimal technology level (that is, independent of the world technology frontier) in the long run.

Furthermore, at steady state, the major macroeconomic variables grow at lower rates in both the regimes in this alternative scenario as compared to the baseline scenario. However, it should be noted that physical capital, aggregate output and consumption grow at a higher rate under imitation regime because imitation economy exhibits a higher rate of technical progress due to the “catch-up effect", which outweighs the erosion effect of technical progress on human capital. Also, per capita income growth rate is very low under both imitation and innovation economies. Per capita income grows slowly under both regimes as aggregate output grows relatively slowly due to low rate of technical progress whereas population grows at a relatively higher rate as parents focus on maximizing fertility and do not invest in education. As a result, growth of per capita income is low. Notably, per capita income growth rate is slightly higher under imitation regime as compared to innovation regime because aggregate output grows at a higher rate under imitation regime due to higher technical progress via the “catch-up effect" as compared to innovation regime. Although, the magnitude of negative erosion effect of technical progress on human capital is higher under imitation regime but the “catch-up effect" outweighs the erosion effect as it has been assumed that imitation is a costless activity.

5 Discussion

A major objective of this paper is to bring a system dynamics perspective to the analytical framework of economics discipline and carry out numerical simulations. A system dynamics model is formulated to study the implications of schooling quality on growth and convergence prospects of a developing economy.

The simulation results reveal that under both the technology regimes, the quality of schooling triggers a child quantity-quality trade-off wherein parents invest in educating their children and bear lesser number of children. Under both the technology regimes, the stylized economy reaches a self-sustaining growth path which depends on rate of human capital accumulation which, in turn, depends on quality of schooling in the long-run. Barely surpassing the threshold of quality of schooling does not warrant a high economic growth rate of the economy under both the regimes. It is numerically substantiated that, instead, quality of schooling should be high enough to counter the erosion effect of technical progress to ensure the economy grows at a higher rate. Also, the simulation results show that an economy which does not invests in education of its future generation is stuck in a low-equilibrium trap.

This model also paves the way for the following future work. Quality of schooling can be endogenized to understand the interrelationships among education, fertility, human capital and technology in a more precise manner. The results of sensitivity analysis reveal that the model structure is sensitive to changes in parameter settings. Using a dynamic parameter setting can be the way forward to improve the efficacy of the model. Also, as a part of future work, it will be useful to enhance the data sources based on macroeconomic variables of different countries to test the historical replicability of the system dynamics model formulated as part of this research work.

Appendix: Solution to Household's Optimization Exercise

The utility function is described as follows:

$$Maximize_{c_{1t},s_t,e_t,n_t}\, u = \, logc_{1t} + \beta_1\, logc_{2,t+1} + \beta_2\, log(h_{t+1}n_t\,);$$

subject to

$$(w_t h_t + r_t \tfrac{K_t}{L_t})(1 - \tau n_t) = \, c_{1t} + s_t + e_t(w_t h_t + r_t \tfrac{K_t}{L_t})n_t$$

$$c_{2,t+1} = \, (1 + r_{t+1})s_t$$

$$h_{t+1} = (\mu + \theta e_t)^{\epsilon} h_t\,, \qquad \epsilon < 1\,.$$

The choice variables are c_{1t}, s_t, e_t and n_t. After substituting for $c_{2,t+1}$ and h_{t+1}, the lagrangean for this problem is formulated as :

$$L = logc_{1t} + \beta_1 \log[(1 + r_{t+1})s_t] + \beta_2 log\, n_t \; + \; \beta_2 \epsilon\, log(\mu + \theta_t e_t) + \beta_2 \epsilon\, log\, h_t \; +$$
$$\psi\left[(w_t h_t + r_t \tfrac{K_t}{L_t})(1 - \tau n_t) - \; c_{1t} - s_t - e_t(w_t h_t + r_t \tfrac{K_t}{L_t}) n_t \right]$$

The first-order conditions are:

$$\frac{\partial L}{\partial c_{1t}} = 0 \;\Leftrightarrow\; \frac{1}{c_{1t}} \text{ - } \psi = 0 \Leftrightarrow c_{1t} = \; \frac{1}{\psi}\,. \tag{A1}$$

$$\frac{\partial L}{\partial s_t} = 0 \;\Leftrightarrow\; \frac{\beta_1}{s_t} \text{ - } \psi = 0 \Leftrightarrow s_t = \; \frac{\beta_1}{\psi}\,. \tag{A2}$$

$$\frac{\partial L}{\partial n_t} = 0 \;\Leftrightarrow\; \frac{\beta_2}{n_t} \quad \text{-} \quad \psi\tau(w_t h_t + r_t \tfrac{K_t}{L_t}) - \; \psi e_t(w_t h_t + r_t \tfrac{K_t}{L_t}) = 0 \;\Leftrightarrow\; \frac{\beta_2}{n_t} \; = \; \psi(e_t + \tau)(w_t h_t + r_t \tfrac{K_t}{L_t}) \quad\Leftrightarrow\quad n_t \; = \; \frac{\beta_2}{\psi(e_t+\tau)(w_t h_t + r_t \frac{K_t}{L_t})} \tag{A3}$$

$$\frac{\partial L}{\partial e_t} = 0 \;\Leftrightarrow\; \frac{\beta_2 \epsilon\, \theta_t}{\mu + \theta_t e_t} \text{ - } \psi n_t(w_t h_t + r_t \tfrac{K_t}{L_t}) \; = 0 \quad\Leftrightarrow\quad n_t \; = \; \frac{\beta_2 \epsilon\, \theta_t}{\psi(\mu + \theta_t e_t)(w_t h_t + r_t \frac{K_t}{L_t})}. \tag{A4}$$

From eqs. (A3) and (A4), the L.H.S can be equated to yield:

$$\mu + \theta_t e_t = \epsilon\, \theta_t (e_t + \tau) \;\Leftrightarrow\; \mu - \; \epsilon\, \theta_t \tau = \theta_t e_t(\epsilon - 1)$$

$$e_t = \; \frac{\mu - \; \epsilon\, \theta_t \tau}{\theta_t(\epsilon - 1)} \; = \frac{\epsilon\, \theta_t \tau \, - \mu}{\theta_t(1 - \epsilon)}$$

Hence, we have:

$$e_t = \begin{cases} 0, & if\; \theta_t < \frac{\mu}{\tau\epsilon}; \\ \frac{\tau\theta_t - \mu}{(1-\epsilon)\theta_t}, & otherwise; \end{cases} \tag{A5}$$

Next, we know that the budget constraint is given by:

$$(w_t h_t + r_t \tfrac{K_t}{L_t})(1 - \tau n_t) = \; c_{1t} + s_t + e_t(w_t h_t + r_t \tfrac{K_t}{L_t}) n_t$$

From eq. (A3), $e_t n_t(w_t h_t + r_t \frac{K_t}{L_t})$ can be expressed as:

$$e_t n_t(w_t h_t + r_t \tfrac{K_t}{L_t}) = \; \frac{\beta_2}{\psi} - \tau n_t(w_t h_t + r_t \tfrac{K_t}{L_t})\,. \tag{A6}$$

Substituting from eqs. (A1), (A2) and (A6), the budget constraint can be expressed as:

$$(w_t h_t + r_t \tfrac{K_t}{L_t}) - \tau n_t(w_t h_t + r_t \tfrac{K_t}{L_t}) = \; \frac{1}{\psi} + \frac{\beta_1}{\psi} + \; \frac{\beta_2}{\psi} - \tau n_t(w_t h_t + r_t \tfrac{K_t}{L_t})\,.$$

which on simplification leads to:

$$\psi = \; \frac{1 + \beta_1 + \beta_2}{(w_t h_t + r_t \frac{K_t}{L_t})}, \tag{A7}$$

whose substitution into eqs. (A1) and (A2) yields:

$$c_{1t} = \frac{(w_t h_t + r_t \frac{K_t}{L_t})}{1+\beta_1+\beta_2};$$

$$s_t = \frac{\beta_1 (w_t h_t + r_t \frac{K_t}{L_t})}{1+\beta_1+\beta_2},$$

Substituting for ψ from eq. (A7) in eq. (A3), yields:

$$n_t = \frac{\beta_2 \epsilon \theta_t}{(1+\beta_1+\beta_2)(\mu + \theta_t e_t)} .$$

This completes the solution to the utility maximization exercise of households.

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