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Abstract

Environmental policies can induce technological changes that lead to lower costs of abatement. We show that a clean technological change is neither necessary nor sufficient for lowering costs of abatement. Even a dirty technological change can do so. By showing that a marginal abatement cost (MAC) of a plant is a ratio of the marginal productivities of the emission-causing input such as a fossil fuel in the production of what is valuable to the firm (here the desired output) and the bad output/emission, the channels by which a technological change can impact the MAC are identified. Depending upon the impact of a technological change on the levels and the slopes of the frontiers of the good and bad outputs, technological change is classified as increasingly clean, decreasingly clean, increasingly dirty, or decreasingly dirty. The MAC schedule of a plant can non-pathologically shift-up (shift-down) – locally or globally – in the case of increasingly clean (increasingly dirty) technological changes, while the MAC will always decrease (increase) in the case of decreasingly clean (decreasingly dirty) technological changes. However, the latter types of technological change are only a local phenomena. The sectoral MAC obtained in a sector with firm-level heterogeneity falls (rises) when all plants in the sector experience decreasingly clean (decreasingly dirty) technological change. However, the impacts of a technological change that is either increasingly clean or increasingly dirty for all plants on the sectoral MAC is ambiguous. A novel non-parametric dual mathematical linear programming approach is developed to estimate the impacts of technological change and changes in the distribution of fixed inputs and emission cap on the sectoral MAC. An application to the Indian thermal power sector reveals that its sectoral MAC rose by 14.5% during 2010-2015. Around 52% of this increase was contributed by technological change and 48% by changes in the fixed inputs – capacity and the managerial input. The former was mainly because most plants in this sector experienced a decreasingly dirty technological change, and the latter was because there was a complementarity between the usage of the two fixed inputs and the variable heat input (coal).

Keywords: Environmental policy-induced technological change, plant-level and sectoral marginal abatement costs, allocative efficiency, by-production approach to modelling emission-generating technologies.

JEL classification codes. Q55, Q50, Q52, D24

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1 Introduction.

Marginal abatement cost (MAC) is an ubiquitous concept in the literature. A production unit needs to employ its marginal abatement cost schedule to achieve its abatement target with a minimum cost. Starting from a baseline level of emission, achievement of an abatement target by a sector in an efficient (cost minimising) manner requires equalisation of the MACs across all production units in that sector. It is well-known that this equalised MAC, which we will call the sectoral MAC, is also the required emission tax that the planner can implement or the equilibrium permit price that will emerge in the market for permits to realise the abatement target set for the sector. The sectoral MAC schedule in conjunction with the schedule for marginal benefits from abatement (that measures the marginal savings from damages from emission when abatement is undertaken) determines the socially optimal level of abatement for the sector. The plant-level and the sectoral MAC will respond to changes in the productive environment of the sector such as changes in the distribution of fixed inputs of production units and the technology.

Policy-oriented literature models, studies, and compares across scenarios that vary with regards to the means and modalities available for tackling the problem of climate change in a least costly manner. In the literature, an important "end" of a successful environmental policy is to induce technological change (ITC) through creating economic incentives for the producing units by lowering their costs of abatement. See, *e.g.,* Downing and White (1986), Grubb et al. (1994), Palmer et al. (1995), Fischer et al. (2003), Goulder and Schneider (1999), Jung and Krutilla (1996), Milliman and Prince (1989), Downing and White (1986), Rosendahl (2004), and Bramoulle and Olson (2005), Goulder and Mathai (2000), Goulder (2004) *etc.*[1] These works compare across environmental policies for the incentives they provide for the development and adoption of technological innovations and study the extent to which ITC promotes greater carbon abatement and impacts its optimal timing. In their analyses, these works almost invariably tend to assume that ITC is a technological change that reduces the total and the marginal costs of abatement of a plant at all levels of abatement. This assumption, which results in higher socially optimal level of abatement and lower permit price, is usually incorporated in the analysis through a reduced form specification of a cost function whose derivative with respect to the technological parameter and whose cross derivative with respect to abatement and the technological parameter (or the derivative of the MAC with respect to the technological parameter) are both negative. Secondly, in these articles, although not modelled explicitly through use of production functions, the ITC is presumed to be a clean one. Thus, in these works, lower total and marginal abatement costs are associated with clean technological

[1] Many of these works and several others have been surveyed effectively in Baker et al. (2008) and Bauman et al. (2008).

changes.

In this work, we find that it is quite possible that a clean technological change may not be the means for achieving the end of lower abatement costs through ITC. Impact of technological change on the total and marginal abatement costs are best studied when technological changes are modelled directly using production functions. Modelled in this way, non-pathological paradoxes may arise – a clean technological change may imply *higher* total and marginal abatement costs at some or all levels of abatement. Moreover, a dirty technological change may imply *lower* total and marginal abatement costs at some or all levels of abatement. It is also possible that the pre and post innovation MAC schedules intersect. If this is true, then computation of economic gains from policy-induced technological changes will have to be more complex and involved than what is usually seen in the above mentioned literature. It may also imply that environmental policies may create incentives for innovations that are fundamentally dirty, albeit can lead to lower total and marginal costs of abatement, which means that they are capable of generating higher abatement and lower permit prices. Are such means for achieving the end, *i.e.,* promotion of ITC that lowers the cost of abatement, perverse or rationalizable?

Works such as Baker et al. (2007), Bauman et al. (2008), and Amir et al. (2008) too argue that there is nothing sacrosanct about clean technological changes leading to lower MACs at all levels of abatement. Baker et al. (2007) provides a diagrammatic example of a downward shift in the graph of a cost function due to a clean technological change that results in the pre and post technological change MAC curves crossing each other, with post-change MAC being lower than the pre-change one at low levels of abatement and higher than the pre-change one at high levels of abatement. In their production-function based approach, Bauman et al. (2008) and Amir et al. (2008) employ traditional comparative static methods to show that clean technological changes can lead to the increases in MAC at some or all levels of abatement.

Our work identifies the exact channels and forces at work for a comprehensive categorisation of the impacts of technological change on the MAC and hence the total abatement cost. This is because our analysis and results are based on a key and fundamental understanding of the concept of the MAC of a plant as a ratio of the marginal productivities of the emission-causing input, such as a fossil fuel, in the production of (i) what creates value for a productive unit (*e.g.,* profits, the scale of production of the good output, etc.) and (ii) its bad output/ emission. In particular, in this study, in line with our empirical work, we assume that the objective of the productive unit is to maximise production of the good output.[2] Two frontiers of an emission-generating technology are relevant – a good-output frontier that defines efficient production of the good output from inputs and an emission/bad-output frontier that defines the minimal generation of emission (the bad output) due to usage of the emission-causing input in production.

A technological change impacts the MAC by impacting both the levels and the slopes of these frontiers of the technology. The latter have the interpretations as the marginal productivities

[2]This is true of the plants in the Indian thermal power sector (most of which are state-owned) on which our empirical analysis is based.

of the emission-causing input in the production of the good and bad outputs, which we denote, respectively, by MP_1 and MP_2.[3] We learn from this analysis that the impacts of a shift in the level and the change in the slope of the emission frontier on MAC due to a technological change are distinct and could work in opposite directions.

Firstly, a clean technological change lowers the bad-output frontier of the technology. Under standard assumptions, this has a negative impact on the MAC. The implicit assumption in the literature that a clean technological change lowers MAC seems to be based on this kind of understanding.

But the shift in the level of the frontier will generally be accompanied by its slope, MP_2, locally or globally decreasing (we call this the case of increasingly clean technological change) or locally increasing (we call this the case of a decreasingly clean technological change). We find that this also has an important bearing on what happens to the MAC. In the former case, the gap between the pre and post technological change frontiers of bad output increases locally with increase in the usage of the emission-causing input. We find that in this case the MAC will tend to rise. The latter case implies that the gap between the two frontiers will decrease locally. In this case, MAC will tend to fall. Similarly too, we can define and the study the implications of an increasingly dirty or a decreasingly dirty technological change on the MAC.

There are thus opposing forces at work influencing the MAC in the case of an increasingly clean technological change. It is possible that such a technological change can lead to a net increase in MAC (locally) at some or (globally) at all levels of abatement. We provide parametric examples that show that this phenomenon is far from pathological and is true under common functional specifications of the technology. On the other hand, in the case of a decreasingly clean technological change, the two forces induced by the change in the level and slope of the emission frontier reinforce each other and will always lead to lower MACs, albeit we find that this phenomenon cannot be globally true over the entire range of abatement. Similarly, an increasingly dirty technological change could lead to lower MACs, while a technological change that is (locally) decreasingly dirty will always lead to an increase in the MAC.

We employ our plant-level analysis to do a novel (non-traditional) comparative static analysis of the impact of technological change on the MAC of a sector comprising of several heterogeneous plants. To see how the sectoral MAC of a sector with many heterogenous units responds to changes in the productive environment we rely on a lemma that says that the sectoral MAC lies in the range of MACs of plants that is obtained at an allocation that is technically efficient but not necessarily allocatively efficient with the same emission cap. Employing this result (a) we prove the negative relationship between the emission cap and the sectoral MAC – we call this the law of cap-induced diminishing sectoral MAC, (b) we show that when fixed inputs of a plant are complementary to the emission-causing input (a variable input) then increases in the fixed inputs will increase the sectoral MAC, and (c) we provide a decomposition of the impact

[3] In our case, where the objective of the plant is assumed to be output maximisation, the MAC is obtained as the ratio $\frac{MP_1}{MP_2}$. If the objective of the firm is profit maximisation, then the MAC can be recomputed as the ratio of marginal profit from the emission-causing input and marginal productivity of this input in the production of emission, $\frac{MP_1 - p_{x_z}}{MP_2}$, where p_{x_z} is the market price of this input.

of a technological change on the sectoral MAC. We show that the total impact of a clean or dirty technological change on the sectoral MAC can be decomposed into (1) the change in the sectoral MAC due to a hypothetical change in the emission cap to the one that the sector can attain with the new technology if the usage of inputs by all plants is unchanged and (2) the change in the sectoral MAC when the sector reverts back to the original cap with the new technology. We find that, if all plants experience a decreasingly clean technological change at the original levels of input usage then (1) and (2) will both be negative and the sectoral MAC will fall. If all plants experience an increasingly clean technological change at the original level of inputs then (1) will be positive, while (2) will be negative and the change in the sectoral MAC will be ambiguous. Similar results are obtained in the cases when all plants experience an increasingly dirty or a decreasingly dirty technological change.

The theory in this paper, which has been developed using a parametric approach to yield sharper and more easily relatable results, was basically motivated and developed from the results we obtained from our empirical exploration that was based on a non-parametric DEA approach. Evaluation of the true performance of an emission-generating sector and checking whether it is in line with the climate change objectives of the policy makers requires specification of a flexible production model. This flexibility is harder to achieve using the parametric approach: it is difficult to come up with flexible enough parametric functional forms of the production functions that can capture any scenario of technological change – increasingly clean/dirty or decreasingly clean/dirty – so that the true nature of the technological change embedded in the data can be revealed. This is relatively more straightforward in the non-parametric DEA approach. In our empirical work based on the Indian thermal power sector we find that for most plants the technological change between 2010 and 2015 has been decreasingly dirty and for some it has been increasingly dirty. The bad-output frontier has shifted up in the local region of inputs for 93% of the plants. At the same time, the slope of this frontier, MP_2, has fallen for 76% of the plants. This change in the technology has contributed to nearly 50% of the 14.5% rise in the sectoral MAC witnessed by this sector in this period. The distribution of fixed inputs – managerial and capacity – has changed in this period. While the distribution of capacity shifted in favour of plants with bigger capacity, the distribution of the managerial input fared poorly in this period. The former tended to increase the sectoral MAC, while the latter tended to decrease it, showing complementarity in our data set between the fixed and the variable (fossil fuel) inputs. In the net, the changes in the distributions of fixed inputs contributed to 48% of the increase in the sectoral MAC. The remaining 2% of the increase in the sectoral MAC came from the technological change in the good-output frontier.

Section 2 briefly reviews the basic by-production model of technology that is employed in this work. Section 3 provides the plant-level and sector-level comparative statics of the MAC along with four key examples. Section 4 presents the non-parametric counterpart of our parametric analysis. Primarily, it develops a methodology to estimate the plant-level and sectoral MAC based on a novel dual linear programming approach. Section 5 presents the data and results from our non-parametric analysis of the Indian thermal power sector. We offer a key discussion of some of our main findings and conclude in Section 6.

2 A simple model of by-production technology.

In our model there is one good output produced by a sector, quantity of which is denoted by $y \in \mathbf{R}_+$. There are N non-emission generating inputs, quantities of which are summarised by the vector $x_o = \langle x_{o_1}, \ldots, x_{o_N} \rangle \in \mathbf{R}^N_+$ and one emission generating input, quantity of which is summarised by $x_z \in \mathbf{R}_+$.[4] There is also one bad output/emission, the amount of which is denoted by $z \in \mathbf{R}_+$. We will also denote $x = \langle x_o, x_z \rangle \in \mathbf{R}^{N+1}_+$ as the quantity vector of all inputs. A production vector is denoted by $\langle x_o, x_z, y, z \rangle$. There are U production units (firms or plants) in the sector. The model presented can be employed for a thermal power sector. In our empirical work, which is based on the Indian thermal power sector, the good output is electricity, the bad output is CO_2 emission, $N = 2$ with capacity and operating availability being the two non-emission generating fixed inputs, and coal measured in heat units is the sole emission-generating variable input.

The by-production technology, developed by Murty and Russell (2002); Murty, Russell and Levkoff (2012) (MRL); and Murty (2015) is defined as $T = T_1 \cap T_2$, where sub-technology T_1 captures standard neo-classical features of a technology using productive inputs to produce a good output.[5] T_1 satisfies convexity and standard neo-classical disposability properties; in particular, it satisfies output free-disposability in good output electricity and input free-disposability in all inputs. Sub-technology T_2, on the other hand, captures production relations in nature between combustion of emission-causing inputs and generation of emissions. T_2 is also convex but violates output free-disposability in emission and input free-disposability in emission-causing inputs. Instead, it satisfies costly disposability of emission and costly disposability of the emission-causing inputs, such as coal. Costly disposability of emission implies that there exists a minimum level of emission generation corresponding to any given amount of the emission-causing input such as coal. Emission generation can be more but not lesser than this level. Costly disposability of the emission-causing input implies that given any fixed level of emission, there exists a maximum amount of the emission-causing input that can generate it. MRL have shown that these disposability assumptions imply that, along the frontier of sub-technology T_1, there is a non-negative trade-off between use of all inputs and production of the good output; while along the frontier of sub-technology T_2, there is a non-negative trade-off between emission-causing inputs and emission generation.

An objective of this work is to study the responsiveness of sectoral MAC to technological change. As pointed out by papers such as Jung and Krutilla (1996), it is important to model

[4] Note, however, that the dual linear programming methodology under a non-parametric specification of the technology that we develop in this work can also be employed in the case where there are more than one emission-causing inputs and when there are several types of emissions.

[5] The by-production approach to modelling emission-generating technologies is motivated by contributions of Nobel laureate Frisch (1965) to the theory of production, whose relevance for modelling emission-generating technologies was first identified by Førsund (1998, 2009, 2018).

firm-level heterogeneity when studying the sector, *e.g.,* firm-level differences provide the economic stimuli for permit trading. In our analysis, we distinguish between fixed inputs, which are assumed to be non-emission causing (such as capacity of a power plant and its managerial input) and variable inputs (for example, coal), which in our model is also the emission-causing one.[6] As in a typical microeconomic production analysis, the levels of the fixed inputs are assumed to be exogenously fixed at any point in time for each plant, but can change over time. Hence, although in the long run when all inputs can vary, all firms have a common by-production technology $T = T_1 \cap T_2$, at any point in time, their respective technologies are conditional on the levels of the fixed inputs employed by them at that point in time and are effectively different. This, in our model, promotes heterogeneity across firms in a sector.

3 Comparative statics of MAC: A parametric analysis.

3.1 A parametric specification of the by-production technology.

Following MRL (2012) parametric representations of the sub-technologies T_1 and T_2 and the overall technology T allowing for technological change are given by

$$\begin{aligned} T_1(\mathcal{A}) &= \{\langle x_o, x_z, y, z\rangle \in \mathbf{R}_+^{N+3} \mid y \leq f(\mathcal{A}, x_o, x_z)\} \\ T_2(\mathcal{B}) &= \{\langle x_o, x_z, y, z\rangle \in \mathbf{R}_+^{N+3} \mid z \geq g(\mathcal{B}, x_z)\} \\ T(\mathcal{A}, \mathcal{B}) &= T_1(\mathcal{A}) \cap T_2(\mathcal{B}) = \{\langle x_o, x_z, y, z\rangle \in \mathbf{R}_+^{N+3} \mid y \leq f(\mathcal{A}, x_o, x_z) \text{ and } z \geq g(\mathcal{B}, x_z)\} \end{aligned} \tag{1}$$

where the production function f (respectively, g) is smooth and concave (respectively, convex) in the inputs with $\frac{\partial f}{\partial x_{o_i}} > 0$ for $i = 1, \ldots, N$; $\frac{\partial f}{\partial x_z} > 0$; and $\frac{\partial g}{\partial x_z} > 0$. Thus, the good output is increasing in the use of both the non-emission and the emission-generating inputs, while the bad output is increasing only in the use of the emission-causing input. We also have $f(\mathcal{A}, x_o, 0) = 0$ and $g(\mathcal{B}, 0) = 0$. The production vector $\langle x_o, x_z, y, z\rangle$ lies on the frontier of technology $T(\mathcal{A}, \mathcal{B})$ or is technically efficient if $y = f(\mathcal{A}, x_o, x_z)$ and $z = g(\mathcal{B}, x_z)$.

There is a local technological progress (respectively, regress) in sub-technology T_1 around input usage $\langle x_o, x_z\rangle$ if the frontier of T_1 shifts up (respectively, down) at that level of input usage, implying more (respectively, less) of the good output can be produced with the same amounts of all the inputs. Similarly, a technological change in sub-technology T_2 is locally clean (respectively, dirty) around input usage x_z if the frontier of T_2 shifts down (respectively, up) at that level of input usage, implying less (respectively, more) emission is generated with the same amount of the emission-causing input. In this model, $\mathcal{A} > 0$ and $\mathcal{B} > 0$ are the technological parameters of sub-technologies T_1 and T_2, respectively. In our analysis we would like to allow for real-life possibilities where these sub-technologies can witness technological changes that are progressive/clean in some regions of input usage and regressive/dirty in others.[7] Hence,

[6]The model can be modified to allow for other non-emission causing variable inputs such as labour.
[7]This is corroborated by our empirical analysis.

if $\frac{\partial f(\mathcal{A},x_o,x_z)}{\partial \mathcal{A}} > 0$ (respectively, $\frac{\partial f(\mathcal{A},x_o,x_z)}{\partial \mathcal{A}} < 0$), then an increase in $\mathcal{A}$ will result in a local technological progress (respectively, regress) in sub-technology T_1 around input usage $\langle x_o, x_z\rangle$. If $\frac{\partial g(\mathcal{B},x_z)}{\partial \mathcal{B}} < 0$ (respectively, $\frac{\partial g(\mathcal{B},x_z)}{\partial \mathcal{B}} > 0$), then an increase in $\mathcal{B}$ is results in a locally clean (respectively, dirty) technological change in sub-technology T_2 around input usage x_z. The opposite is true if there is a decrease in $\mathcal{A}$ or $\mathcal{B}$, [8] Hence, without loss of generality, in this paper, we will assume that technological change implies increase in $\mathcal{A}$ or $\mathcal{B}$ or both.

The above features of functions f and g will be maintained throughout the analysis.

3.2 Parametric approach to the derivation of MAC of a plant and its interpretation as the shadow price of emission.

In this work, we assume that the objectives of a firm/plant/unit and the sector in which it operates are to maximise their respective outputs. Consider a production vector $v = \langle x_o, x_z, y, z\rangle$ lying on the frontier of the technology of a plant that is conditional on its levels of usage of the fixed inputs and apply the inverse function theorem to invert function g in (1) to obtain $x_z = h(\mathcal{B}, z) \equiv g^{-1}(\mathcal{B}, z)$. The function h gives the maximum amount of the emission-causing input that can generate z amount of CO_2. Now plug this function into function f to obtain $y = f(\mathcal{A}, x_o, h(\mathcal{B}, z))$. The MAC of the plant at the frontier point v of its technology is then obtained as

$$MAC(\mathcal{A}, \mathcal{B}, x_o, x_z) = \frac{\partial y}{\partial z} = \frac{\partial f(\mathcal{A}, x_o, x_z)}{\partial x_z}\frac{\partial h(\mathcal{B}, z)}{\partial z} = \frac{\frac{\partial f(\mathcal{A},x_o,x_z)}{\partial x_z}}{\frac{\partial g(\mathcal{B},x_z)}{\partial x_z}} =: \frac{MP_1(\mathcal{A}, x_o, x_z)}{MP_2(\mathcal{B}, x_z)} \tag{2}$$

Thus, as seen in (2), the MAC is a ratio of the marginal productivities of the emission-causing input in the production of the good and bad outputs, denoted by $MP_1 = \frac{\partial f}{\partial x_z}$ and $MP_2 = \frac{\partial g}{\partial x_z}$, respectively. It is obtained by first considering the extent to which the emission-causing input has to be reduced by the plant when emission is sought to be reduced by a unit (given by $dx_z = \frac{\partial h(\mathcal{B},z)}{\partial z} = \frac{1}{\frac{\partial g(\mathcal{B},x_z)}{\partial x_z}}$) and then noting the impact of this reduction in the emission-causing input on the good output of the plant (given by $\frac{\partial f(\mathcal{A},x_o,x_z)}{\partial x_z}dx_z$). Thus, the formula for MAC in (2) is reflective of the fact that a reduction in emission of an efficiently operated plant necessitates reduction in the use of the emission-causing input, which in turn reduces the production of the good output.

The above definition of MAC can be related to the standard MAC schedule of plant once a base-line level of emission for the plant is defined, say $\bar{z}$. The cost of abatement for this plant is then defined in a standard manner as the reduction in its good output plant due to emission abatement (denoted by θ) starting from its baseline emission.

$$C(\mathcal{A}, \mathcal{B}, x_o, \theta) = f(\mathcal{A}, x_o, h(\mathcal{B}, \bar{z})) - f(\mathcal{A}, x_o, h(\mathcal{B}, \bar{z} - \theta)) \ \forall\ \theta \in [0, \bar{z}].$$

[8] For example, a decrease in $\mathcal{B}$ when $\frac{\partial g(\mathcal{B},x_z)}{\partial \mathcal{B}} < 0$ implies a locally dirty technological progress, while a decrease in $\mathcal{B}$ when $\frac{\partial g(\mathcal{B},x_z)}{\partial \mathcal{B}} > 0$ implies a locally clean technological progress.

The traditional MAC schedule for the plant is then obtained using (2) as

$$\begin{aligned} m(\mathcal{A},\mathcal{B},x_o,\theta) = \frac{\partial C(\mathcal{A},\mathcal{B},x_o,\theta)}{\partial\theta} &= \frac{\partial f(\mathcal{A},x_o,x_z)}{\partial x_z}\frac{\partial h(\mathcal{B},\bar{z}-\theta)}{\partial z} \\ &= \frac{MP_1(\mathcal{A},x_o,x_z)}{MP_2(\mathcal{B},x_z)} = MAC(\mathcal{A},\mathcal{B},x_o,x_z) \end{aligned} \tag{3}$$

where $x_z = h(\mathcal{B},\bar{z}^u-\theta)$. Hence $MAC(\mathcal{A},\mathcal{B},x_o,x_z)$ derived in (2) can be interpreted as the MAC for plant when its abatement is equal to $\theta = \bar{z} - g(\mathcal{B},x_z)$ where $x_z \in [0, h(\mathcal{B},\bar{z})]$.

The concept of the MAC derived in (2) can also be interpreted as the shadow price of emission for the plant as it turns out to be the Lagrange multiplier of the emission constraint z in the problem.

$$\mathrm{y}(\mathcal{A},\mathcal{B},x_o,z) = \max_{\hat{x}_z}\{f(\mathcal{A},x_o,\hat{x}_z) \mid z \geq g(\mathcal{B},\hat{x}_z)\} \tag{4}$$

Suppose x_z is the solution. Then $z = g(\mathcal{B},x_z)$, and employing the envelope theorem and the first-order condition of the Problem (4), the Lagrange multiplier of the problem is

$$\Gamma(\mathcal{A},\mathcal{B},x_o,z) = \frac{\partial \mathrm{y}(\mathcal{A},\mathcal{B},x_o,z)}{\partial z} = \frac{\frac{\partial f(\mathcal{A},x_o^u,x_z)}{\partial x_z}}{\frac{\partial g(\mathcal{B},x_z)}{\partial x_z^u}} = \frac{MP_1(\mathcal{A},x_o,x_z)}{MP_2(\mathcal{B},x_z)} = MAC(\mathcal{A},\mathcal{B},x_o,x_z) \tag{5}$$

3.3 The impact of technological changes on plant-level MAC.

Suppose under sub-technology $T_2(\mathcal{B})$ the plant was employing x_z level of the emission-causing input and generating $z = g(\mathcal{B},x_z)$ amount of emission, *i.e.,* its level of abatement is $\theta = \bar{z} - z$, where $z \leq \bar{z}$. Then the MAC of the plant is obtained from (3) as

$$m(\mathcal{A},\mathcal{B},x_o,\bar{z}-z) = \frac{MP_1(\mathcal{A},x_o,h(\mathcal{B},z))}{MP_2(\mathcal{B},h(\mathcal{B},z))} \tag{6}$$

As the level of emission z of the plant decreases from the baseline level $\bar{z}$ to 0, its abatement level increases from 0 to $\bar{z}$.

Now suppose the technological parameter of sub-technology T_2 increases from $\mathcal{B}$ to $\bar{\mathcal{B}}$ and shifts the frontier of sub-technology T_2. As a result, depending upon whether the increase in $\mathcal{B}$ leads to a clean or dirty technological change, the plant will emit less or more than z amount of the emission at the original level of input x_z.[9] Panel (a) of Figure 1 shows the case of a clean technological change where the plant emits less, while Panel (b) shows the case of a dirty technological change where it emits more than z amount of emission at the original level of input x_z. This implies that if the technological change is clean then, to maintain its original level of abatement θ under the new sub-technology $T_2(\bar{\mathcal{B}})$, the plant will have to use more than x_z amount of the emission-causing input, while if the technological change is dirty, it will need to use less than x_z amount of the input. Panel (a) of Figure 1 shows that the usage of the

[9]The plant will emit less (respectively, more) with increase in $\mathcal{B}$ if $\frac{\partial g(\mathcal{B},x_z)}{\partial \mathcal{B}} < 0$ (respectively, $\frac{\partial g(\mathcal{B},x_z)}{\partial \mathcal{B}} > 0$).

input has to increase from x_z to $\hat{x}_z$, while Panel (b) shows that the usage of the input has to decrease from x_z to $\bar{x}_z$ to maintain the original level of abatement. The change in the usage of the emission-causing input to maintain the original level of abatement by the plant in the wake of the technological change in T_2 is captured by the derivative $\frac{\partial h(\mathcal{B},z)}{\partial \mathcal{B}}$.

The theorem below, which shows the impact of technological changes in T_1 and T_2 on the plant's MAC, follows directly from (6).

Theorem 1 *The rates of proportional change in the MAC of a plant with respect to the technological parameters $\mathcal{B}$ and $\mathcal{A}$ are given by*

$$\underbrace{\frac{\partial \ln m\left(\mathcal{A},\mathcal{B},x_o,\bar{z}-z\right)}{\partial \mathcal{B}}}_{\frac{\dot{m}_{\mathcal{B}}}{m}(\mathcal{A},\mathcal{B},x_o,\bar{z}-z)} =$$

$$-\underbrace{\frac{\partial \ln MP_2\left(\mathcal{B},x_z\right)}{\partial \mathcal{B}}}_{\frac{\dot{MP}_{2_{\mathcal{B}}}}{MP_2}(\mathcal{B},x_z)} - \underbrace{\frac{\partial \ln MP_2\left(\mathcal{B},x_z\right)}{\partial x_z}}_{\frac{\dot{MP}_{2_{x_z}}}{MP_2}(\mathcal{B},x_z)} \underbrace{\frac{\partial h(\mathcal{B},z)}{\partial \mathcal{B}}}_{h_{\mathcal{B}}(\mathcal{B},z)} + \underbrace{\frac{\partial \ln MP_1\left(\mathcal{A},x_o,x_z\right)}{\partial x_z}}_{\frac{\dot{MP}_{1_{x_z}}}{MP_1}(\mathcal{A},x_o,x_z)} \underbrace{\frac{\partial h(\mathcal{B},z)}{\partial \mathcal{B}}}_{h_{\mathcal{B}}(\mathcal{B},z)},$$

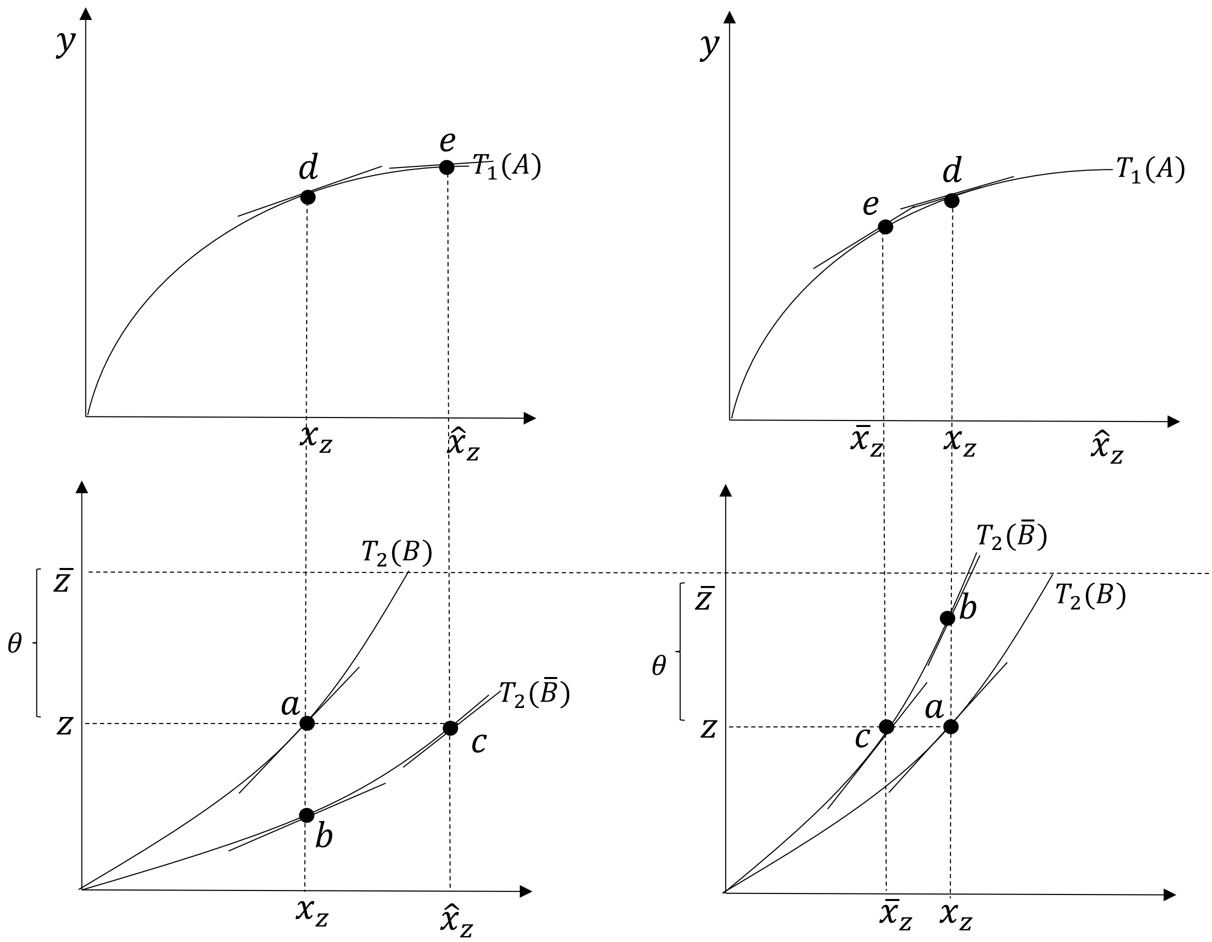
$$\underbrace{\frac{\partial \ln m\left(\mathcal{A},\mathcal{B},x_o,\bar{z}-z\right)}{\partial \mathcal{A}}}_{\frac{\dot{m}_{\mathcal{A}}}{m}(\mathcal{A},\mathcal{B},x_o,\bar{z}-z)} = \underbrace{\frac{\partial \ln MP_1\left(\mathcal{A},x_o,x_z\right)}{\partial \mathcal{A}}}_{\frac{\dot{MP}_{1_{\mathcal{A}}}}{MP_1}(\mathcal{A},x_o,x_z)}$$

where $x_z = h(\mathcal{B},z)$.

The theorem above implies that the proportional change in the MAC of the plant due to a technological change in sub-technology T_2 is nothing but the difference in the percentage changes in MP_1 and MP_2 that it induces when the level of abatement is held fixed.

The percentage change in MP_2 induced by the technological change can inturn be decomposed into two parts: (i) $\frac{\dot{MP}_{2_{\mathcal{B}}}}{MP_2}(\mathcal{B},x_z)$ that measures the rate of proportional change in MP_2 due to change in $\mathcal{B}$ when the emission-causing input usage is unchanged at x_z. In Panels (a) and (b) of Figure 1 it is the percentage change in MP_2 (slope of function g or frontier of T_2) as we move from point a to b. (ii) The term $\frac{\dot{MP}_{2_{x_z}}}{MP_2}(\mathcal{B},x_z)\, h_{\mathcal{B}}(\mathcal{B},z)$ that measures the percentage change in MP_2 due to the change in usage of the emission causing input that is now required to achieve the original level of abatement because of the shift in the emission frontier induced by the technological change in T_2. Thus, in Figure 1, this is the percentage changes in MP_2 as we move from point b to c, where the emission level is held fixed at z under both the old and new T_2 sub-technologies.

The term $\frac{\dot{MP}_{1_{x_z}}}{MP_1}(\mathcal{A},x_o,x_z)\, h_{\mathcal{B}}(\mathcal{B},z)$ measures the percentage change in MP_1 as a result of the change in usage of the emission causing input that is required under the technological change to maintain the original level of abatement. Thus in Figure 1 it is the percentage change in MP_1 (slope of function f or the frontier of T_1) as we move from point d to e.



a. Clean technological change

b. Dirty technological change

Figure 1

Theorem 1 also states that the proportional change in the plant's MAC due to a technological change in sub-technology T_1 is just the proportional change in the MP_1 induced by change in the technological parameter $\mathcal{A}$.

A technological change in any sub-technology implies both a change in the slope and the level of its frontier. Theorem 1 highlights the difference in the way technological changes in sub-technologies T_1 and T_2 affect the plant's MAC. A technological change in sub-technology T_1 impacts the MAC only because of the change in the slope (MP_1) that it induces at every level of the emission-causing input: Theorem 1 shows that $\frac{\dot{m}_{\mathcal{A}}}{m}$ depends only on $\dot{MP}_{1_{\mathcal{A}}}$. The impact of the technological change on the level of the frontier of T_1 seems to have no bearing on the direction of change in MAC of the plant.

On the other hand, a technological change in sub-technology T_2 impacts the MAC because of its impact on both the slope (MP_2) and the level of the frontier of T_2. Thus, $\frac{\dot{m}_{\mathcal{B}}}{m}$ depends not only on $\frac{\dot{MP}_{2_{\mathcal{B}}}}{MP_2}$ but also on $\frac{\dot{MP}_{2_{x_z}}}{MP_2}(\mathcal{B}, x_z)\, h_{\mathcal{B}}(\mathcal{B}, z)$ as the change in the level of the frontier due to the technological change implies that at the original level of the emission-causing input the original level of abatement cannot be sustained by the new technology.

Remark 2 *The signs of both* $-\frac{\dot{MP}_{2_{x_z}}}{MP_2}(\mathcal{B}, x_z)\, h_{\mathcal{B}}(\mathcal{B}, z)$ *and* $\frac{\dot{MP}_{1_{x_z}}}{MP_1}(\mathcal{A}, x_o, x_z)\, h_{\mathcal{B}}(\mathcal{B}, z)$ *are the* *same. If technological change in T_2 is clean (respectively, dirty) at input level x_z then they are both non-positive (respectively, non-negative).*

This remark follows because

$$\frac{\partial MP_2(\mathcal{B}, x_z)}{\partial x_z} = \frac{\partial^2 g(\mathcal{B}, h(\mathcal{B}, z))}{\partial x_z^2} \geq 0, \quad \frac{\partial MP_1(\mathcal{A}, x_o, h(\mathcal{B}, x_z))}{\partial x_z} = \frac{\partial^2 f(\mathcal{A}, x_o, x_z)}{\partial x_z^2} \leq 0,$$
$$\frac{\partial h(\mathcal{B}, z)}{\partial \mathcal{B}} = -\frac{\frac{\partial g(\mathcal{B}, h(\mathcal{B}, z))}{\partial \mathcal{B}}}{\frac{\partial g(\mathcal{B}, h(\mathcal{B}, z))}{\partial x_z}} = \frac{-\partial g(\mathcal{B}, x_z)/\partial \mathcal{B}}{MP_2(\mathcal{B}, x_z)} \tag{7}$$

The signs of the derivatives of MP_2 and MP_1 with respect to x_z follow from our assumptions on functions f and g. Further, if the technological change in sub-technology T_2 is clean ($\partial g(\mathcal{B}, x_z)/\partial \mathcal{B} < 0$) then the sign of the derivative of h with respect to $\mathcal{B}$ is positive, and it is negative if the technological change is dirty ($\partial g(\mathcal{B}, x_z)/\partial \mathcal{B} > 0$).

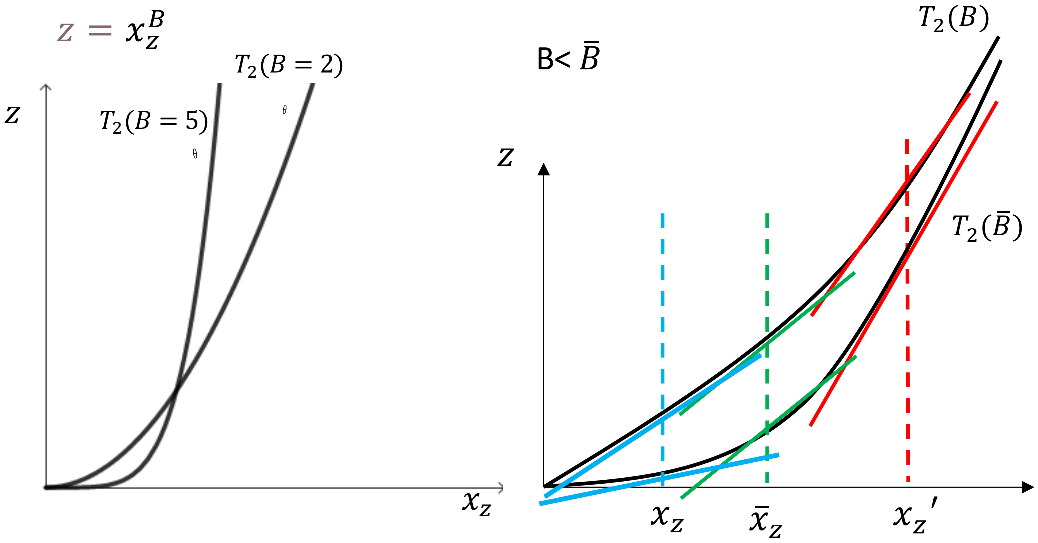
We next classify the different types of technological changes in sub-technology T_2 that are possible based on the types of shift in the level and the types of change in the slope of its frontier that they induce:

Definition 3 *Starting from $T_2(\mathcal{B})$, a technological change in sub-technology T_2 due to an increase in $\mathcal{B}$ is*

- *increasingly clean at x_z if it is locally clean at x_z and* $\frac{\partial MP_2(\mathcal{B}, x_z)}{\partial \mathcal{B}} < 0$.
- *decreasingly clean at x_z if it is locally clean at x_z and* $\frac{\partial MP_2(\mathcal{B}, x_z)}{\partial \mathcal{B}} > 0$.
- *increasingly dirty at x_z if it is locally dirty at x_z and* $\frac{\partial MP_2(\mathcal{B}, x_z)}{\partial \mathcal{B}} > 0$.
- *decreasingly dirty at x_z if it is locally dirty at x_z and* $\frac{\partial MP_2(\mathcal{B}, x_z)}{\partial \mathcal{B}} < 0$.

The technological change is globally increasingly/decreasingly clean or globally increasingly/decreasingly dirty if it is increasingly/decreasingly clean or increasingly/decreasingly dirty at all $x_z \geq 0$, respectively.

When a technological change is clean and involves a decrease in MP_2 at x_z level of the emission-causing input, the gap in the pre and post technological change frontiers of T_2 will locally increase with increase in usage of the emission-causing input, *i.e.,* the reduction in emission due to the technological change increases with increase in the usage of the emission-causing input in a local neighbourhood of x_z, indicating that the technology is becoming even more clean in this local neighbourhood with increase in usage of the input. Hence, we call such a technological change increasingly clean. However, if MP_2 decreases at x_z, then the gap in the pre and post technological change frontiers of T_2 will locally decrease with increase in usage of the emission-causing input, indicating that the technology is becoming less clean with increase in usage of the emission-causing input in a local neighbourhood of x_z. Hence, we call such a technological change decreasingly clean. Similarly, we can also explain the increasingly and decreasingly dirty technological changes. In the former case, a dirty technological change becomes locally even more dirty with increase in usage of the emission-causing input, while in the latter case it becomes locally less dirty.



a. b.

Figure 2

In Panel (a) (respectively, Panel (b)) of Figure 1, MP_2 decreases (respectively, increases) as we move from point a to point b due to a clean (respectively, dirty) technological change in T_2 ($\bar{\mathcal{B}} > \mathcal{B}$) when the level of the emission-causing input is held fixed at x_z. Hence, this technological change is increasingly clean (respectively, increasingly dirty) technological change at x_z level of the emission-causing input. Figure 2 shows examples of changes in sub-technology T_2 that are increasingly clean to begin with and then become decreasingly clean. In Panel (a), where the frontier of T_2 is given by $z = g(\mathcal{B}, x_z) = x_z^{\mathcal{B}}$, it can be verified that the technological change is clean when $x_z \in [0, 1]$ after which it is dirty. The pre and post technological change frontiers of T_2 cross each other at $x_z = 1$. In the region where the technological change is clean, we can verify that it is increasingly clean when $x_z \in [0, e^{\frac{1}{\mathcal{B}}})$ and decreasingly clean when $x_z \in (e^{\frac{1}{\mathcal{B}}}, 1]$. Panel (b) illustrates the case of a globally clean technological change (T_2 changes from $T_2(\mathcal{B})$ to $T_2(\bar{\mathcal{B}})$ with $\mathcal{B} < \bar{\mathcal{B}}$), where the technological change is increasingly clean in the range of input usage $[0, \bar{x}_z]$, after which it is decreasingly clean. In the range $[0, \bar{x}_z]$, MP_2 falls due to the technological change, but beyond this range, MP_2 increases due to the technological change.

If a technological progress is increasingly clean at x_z level of input usage, then the marginal productivity of the emission-causing input in producing the emission, MP_2, falls, making it increasingly harder to abate emission at that point – a unit reduction in emission will require a much greater reduction in the usage of the emission-causing input leading to a greater loss in the good output and hence a higher MAC at x_z level of input usage. Hence, in Theorem 1, $\frac{\dot{MP}_{2_{\mathcal{B}}}}{MP_2}(\mathcal{B}, x_z) < 0$ has an increasing effect on the MAC. The converse argument holds if the technological progress is decreasingly clean at x_z. If MP_2 increases at x_z when there is a clean technological change, then a unit reduction in emission requires only a small reduction in the use of the emission-causing input and hence a smaller reduction in the production of the good output. This implies a lower MAC at x_z level of input usage. Similar arguments can be made for the cases of the decreasingly and increasingly dirty technological changes.

The corollary to Theorem 1 below provides a comprehensive list of possible ways in which a technological change in T_2 can impact a plant's MAC.

Corollary 4 *The impact of a technological change in T_2 on the MAC of a plant in a local neighbourhood around x_z can be classified as follows employing Theorem 1 and Remark 2:*

(i) If the technological change is decreasingly clean at x_z, then

$$-\frac{\dot{MP}_{2_{\mathcal{B}}}}{MP_2}(\mathcal{B}, x_z) < 0 \text{ and } -\frac{\dot{MP}_{2_{x_z}}}{MP_2}(\mathcal{B}, x_z)\, h_{\mathcal{B}}(\mathcal{B}, z) + \frac{\dot{MP}_{1_{x_z}}}{MP_1}(\mathcal{A}, x_o, x_z)\, h_{\mathcal{B}}(\mathcal{B}, z) \leq 0$$

so that the plant's MAC will fall at x_z

(ii) If the technological change is decreasingly dirty at x_z, then

$$-\frac{\dot{MP}_{2_{\mathcal{B}}}}{MP_2}(\mathcal{B}, x_z) > 0 \text{ and } -\frac{\dot{MP}_{2_{x_z}}}{MP_2}(\mathcal{B}, x_z)\, h_{\mathcal{B}}(\mathcal{B}, z) + \frac{\dot{MP}_{1_{x_z}}}{MP_1}(\mathcal{A}, x_o, x_z)\, h_{\mathcal{B}}(\mathcal{B}, z) \geq 0$$

so that the plant's MAC will rise at x_z

(iii) If the technological change is increasingly clean at x_z, then

$$-\frac{\dot{MP}_{2_{\mathcal{B}}}}{MP_2}(\mathcal{B}, x_z) > 0 \text{ and } -\frac{\dot{MP}_{2_{x_z}}}{MP_2}(\mathcal{B}, x_z)\, h_{\mathcal{B}}(\mathcal{B}, z) + \frac{\dot{MP}_{1_{x_z}}}{MP_1}(\mathcal{A}, x_o, x_z)\, h_{\mathcal{B}}(\mathcal{B}, z) \leq 0$$

and the sign of the change in the plant's MAC will be ambiguous.

(iv) If the technological change is increasingly dirty at x_z, then

$$-\frac{\dot{MP}_{2_{\mathcal{B}}}}{MP_2}(\mathcal{B}, x_z) < 0 \text{ and } -\frac{\dot{MP}_{2_{x_z}}}{MP_2}(\mathcal{B}, x_z)\, h_{\mathcal{B}}(\mathcal{B}, z) + \frac{\dot{MP}_{1_{x_z}}}{MP_1}(\mathcal{A}, x_o, x_z)\, h_{\mathcal{B}}(\mathcal{B}, z) \geq 0$$

and the sign of the change in the plant's MAC will be ambiguous.

Can a globally clean technological change be globally decreasingly clean or can a globally dirty technological change be globally decreasingly dirty? The following proposition answers this question in the negative.

Proposition 5 *Suppose $MP_2(\mathcal{B}, 0) \neq MP_2(\bar{\mathcal{B}}, 0)$ whenever $\mathcal{B} \neq \bar{\mathcal{B}}$. Starting from $T_2(\mathcal{B})$, if a technological change in sub-technology T_2 is*

(i) globally clean then there exists $\delta > 0$ such that it is increasingly clean at all $x_z \in [0, \delta)$.

(ii) globally dirty then there exists $\delta > 0$ such that it is increasingly dirty at all $x_z \in [0, \delta)$.

The proposition implies that a globally clean technological change in T_2 must start out being increasingly clean. It can turn decreasingly dirty at higher levels of usage of the emission-causing input. Similarly, a globally dirty technological progress in T_2 must start out being increasingly dirty and can become decreasingly dirty later.

3.4 Some special cases of Theorem 1.

We now provide four key examples to demonstrate the varied ways in which a technological change in sub-technology T_2 can impact the MAC of a plant.

In all the examples below, sub-technology T_1 is Cobb-Douglas: $y = f(\mathcal{A}, x_o, x_z) = \mathcal{A} x_z^{\alpha} x_o^{1-\alpha}$, $\alpha \in (0,1)$. In Examples 1 and 3, the frontier of sub-technology T_2 is linear in x_z. The linearity of the frontier of T_2 in x_z implies that the term $\frac{\dot{MP}_{2_{x_z}}}{MP_2}(\mathcal{B}, x_z)\, h_{\mathcal{B}}(\mathcal{B}, z)$ in (7) is zero. Hence, it follows from Theorem 1 and Corollary 4 that the proportional change in the plant's MAC due to a technological change will depend only on the relative magnitudes of the terms $-\frac{\dot{MP}_{2_{\mathcal{B}}}}{MP_2}(\mathcal{B}, x_z)$ and $\frac{\dot{MP}_{1_{x_z}}}{MP_1}(\mathcal{A}, x_o, x_z)\, h_{\mathcal{B}}(\mathcal{B}, z)$.

Example 1

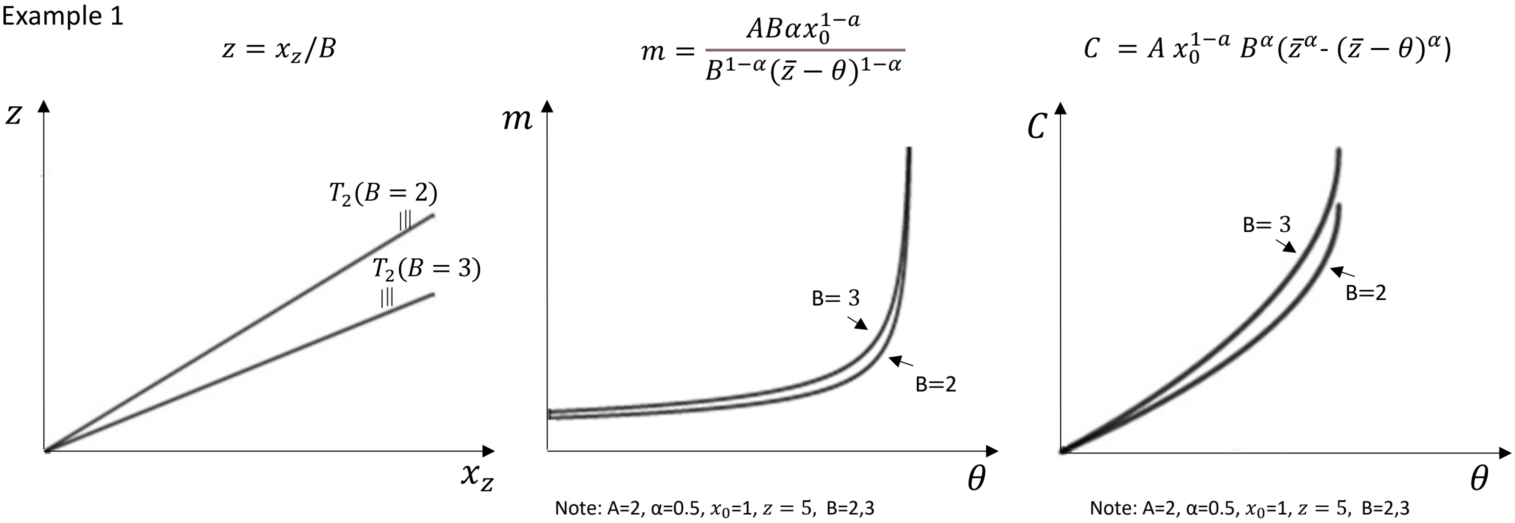


Figure 3

In particular, in Example 1, as in Amir et al. (2008) and Bauman et al. (2008), we have $z = g(\mathcal{B}, x_z) = \frac{x_z}{\mathcal{B}}$, $\mathcal{B} > 1$. This implies the following:

$$\frac{\partial g(\mathcal{B}, x_z)}{\partial \mathcal{B}} = -\frac{x_z}{\mathcal{B}^2} < 0, \quad MP_2(\mathcal{B}, x_z) = \frac{1}{\mathcal{B}}, \quad \text{and} \quad \dot{MP}_{2_{\mathcal{B}}} = \frac{\partial MP_2(\mathcal{B}, x_z)}{\partial \mathcal{B}} = -\frac{1}{\mathcal{B}^2} < 0.$$

Hence, this is clearly an example of a globally increasingly clean technological change, where an increase in $\mathcal{B}$ shifts the frontier of T_2 down and reduces MP_2 at every level of the emission-causing input. It can be readily verified that, in this example, Theorem 1 implies

$$\frac{\dot{m}_{\mathcal{B}}}{m}(\mathcal{A}, \mathcal{B}, x_o, \bar{z} - z) = -\frac{\dot{MP}_{2_{\mathcal{B}}}}{MP_2}(\mathcal{B}, x_z) + \frac{\dot{MP}_{1_{x_z}}}{MP_1}(\mathcal{A}, x_o, x_z)\, h_{\mathcal{B}}(\mathcal{B}, z) = \frac{\alpha}{\mathcal{B}} > 0.$$

Thus, we have a non-pathalogical example of a case where technological change in clean and increasingly so but where the MAC shifts up globally and so does the total abatement cost. See Figure 3, which corresponds to this example, where the impacts of an increase in $\mathcal{B}$ are traced out.

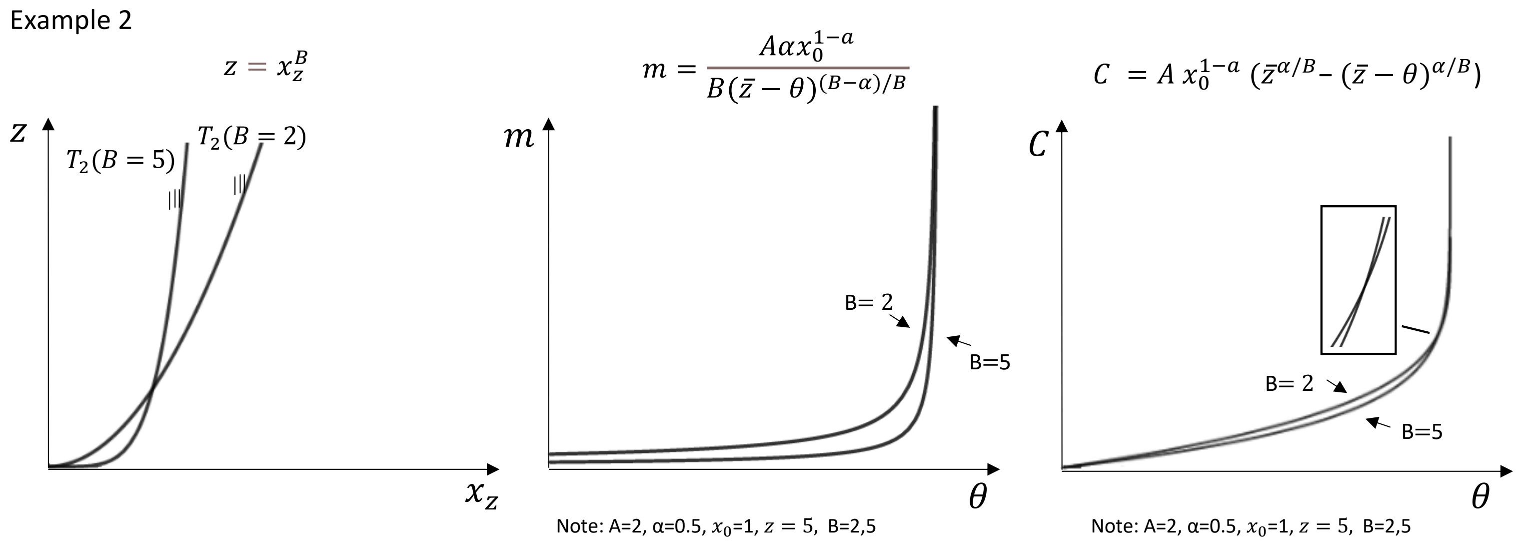


Figure 4

Example 2 demonstrates a case where the technological change in sub-technology T_2 is decreasingly clean in a local region. Here T_2 is specified as in Panel (a) of Figure 2, which was discussed in Section 3.3. As shown in Figure 4, the MAC and cost schedules shift down when $\mathcal{B}$ increases from 2 to 5 in this case. In this case, Theorem 1 implies that, in the decreasingly clean region of the technological change, the impacts on MAC due to the downward shift in the frontier of T_2 and the increase in its slope, MP_2, reinforce each other. In the increasingly clean region, though the two impacts work in opposite direction, the impact due to the downward shift in the frontier is weaker and is offset by the impact due to the increase in the slope.

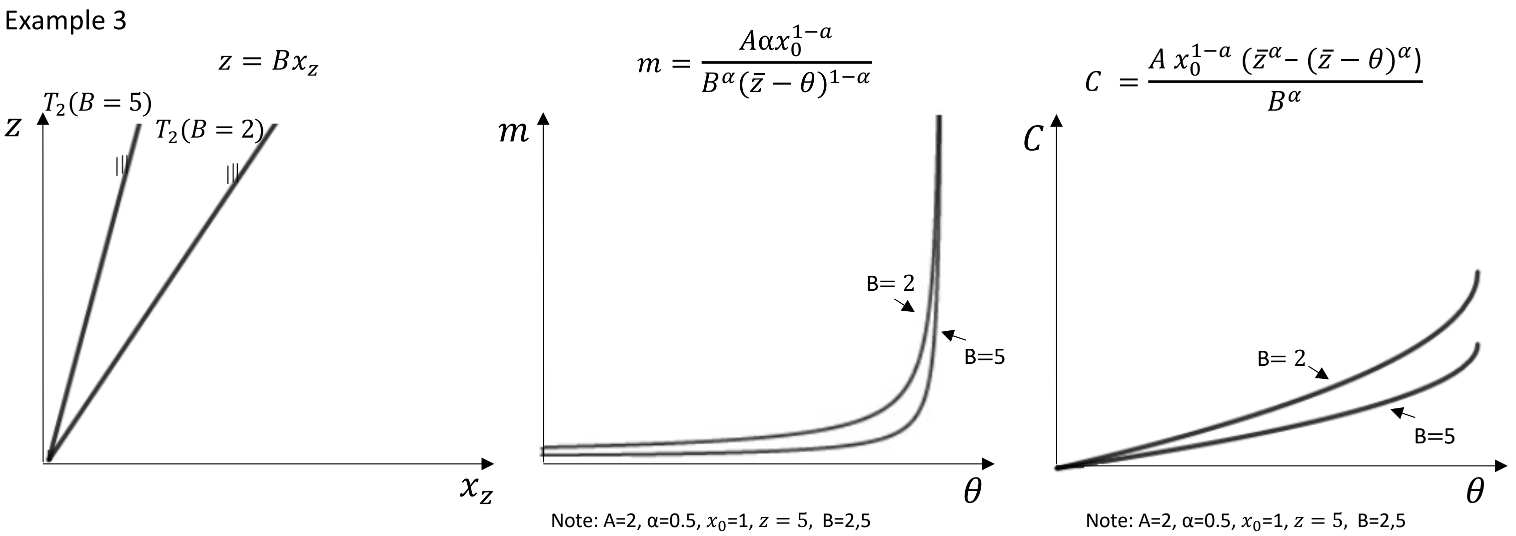


Figure 5

Example 3, which is illustrated in Figure 5, shows that even an increasingly dirty technological change can result in downward shifts of the total abatement cost and the MAC schedules. Here $z = g(\mathcal{B}, x_z) = \mathcal{B}x_z$ with $\mathcal{B} > 1$, so that $MP_2(\mathcal{B}, x_z) = \mathcal{B}$, $\frac{\partial g(\mathcal{B},x_z)}{\partial \mathcal{B}} = x_z > 0$, and $\frac{\partial MP_2(\mathcal{B},x_z)}{\partial \mathcal{B}} = 1 > 0$ for all $x_z > 0$. Hence, the proportional change in MAC in this case is negative:

$$\frac{\dot{m}_{\mathcal{B}}}{m}\left(\mathcal{A},\mathcal{B},x_o,\bar{z}-z\right) \;=\; -\frac{\dot{MP}_{2_{\mathcal{B}}}}{MP_2}\left(\mathcal{B},x_z\right)+\frac{\dot{MP}_{1_{x_z}}}{MP_1}\left(\mathcal{A},x_o,x_z\right)h_{\mathcal{B}}(\mathcal{B},z) \;=\; -\frac{\alpha}{\mathcal{B}}<0$$

This leads us to questions such as whether environmental policy can make it worthwhile for firms and the society to invest in such types of dirty technological innovations. It seems that such technological changes will lead to greater abatement and lower permit prices.

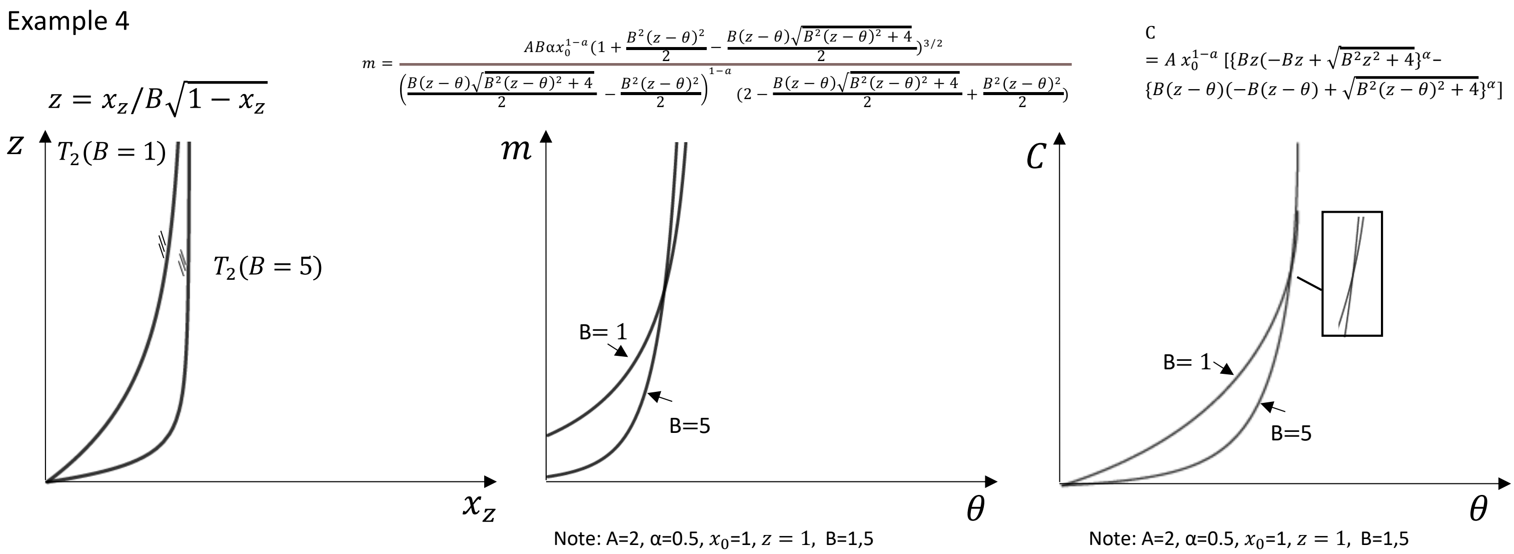


Figure 6

In Example 4, our last example, which is illustrated in Figure 6, the sub-technology T_2 is specified by the function $z = g(\mathcal{B}, x_z) = \frac{x_z}{\mathcal{B}\sqrt{1-x_z}}$ with $\mathcal{B} > 1$. This technology is well-defined in the range $x_z \in [0, 1]$. Since $\frac{\partial g}{\partial \mathcal{B}} < 0$ in this range, technological change implied by an increase in $\mathcal{B}$ will be globally clean. Since $MP_2 = \frac{1}{B\sqrt{1-x}}\left[1+\frac{x}{2(1-x)}\right]$, it can be seen that $\frac{\partial MP_2}{\partial \mathcal{B}} < 0$. Hence, the technological change is an increasingly clean one. However, in this case, unlike in Example 1, the post technological change MAC lies below the pre technological change initially and then lies above it. The two curves cross each other.

3.5 Parametric characterisation of allocative efficiency.

We now move from a plant-level to a sector-level characterisation of the MAC. A sector comprises of many heterogenous plants. When a sector achieves allocative efficiency under a pre-stipulated sectoral emission cap, then there is no redistribution of sectoral emission among firms that can further increase the amount of electricity generated by the sector. Thus a sector comprising of U plants with distribution of the fixed inputs $\langle x_o^1, \ldots, x_o^U \rangle$, technological change parameters $\mathcal{A}$ and $\mathcal{B}$, and emission cap z achieves allocative efficiency at the solution of the following problem that maximises sectoral production of the good output subject to the constraint imposed by the sectoral emission cap:

$$\mathcal{Y}\left(\mathcal{A},\mathcal{B},x_o^1,\ldots,x_o^U,z\right) := \max_{\{x_z^u\}_{u=1}^U}\left\{\sum_{u=1}^U f\left(\mathcal{A},x_o^u,x_z^u\right) \;\middle|\; \sum_{u=1}^U g\left(\mathcal{B},x_z^u\right) \;\le\; z\right\}. \tag{8}$$

Denote the solution functions that give the optimal levels of the good output, emission, and the emission-causing input for each plant u as well as the value of the Lagrange multiplier at the optimum, respectively, as

$$\mathcal{Y}^u = \mathcal{Y}^u\left(\mathcal{A},\mathcal{B},x_o^1,\ldots,x_o^U,z\right),\; \mathfrak{Z}^u = \mathfrak{Z}^u\left(\mathcal{A},\mathcal{B},x_o^1,\ldots,x_o^U,z\right),\; \mathfrak{X}_z^u = \mathfrak{X}_z^u\left(\mathcal{A},\mathcal{B},x_o^1,\ldots,x_o^U,z\right),$$
$$\Psi = \Psi\left(\mathcal{A},\mathcal{B},x_o^1,\ldots,x_o^U,z\right) \tag{9}$$

Assuming an interior solution, the envelope theorem and the first-order conditions of problem (8) together with (5) and (9) imply that, for all $u = 1,\ldots,U$, we have

$$\begin{aligned} MAC\left(\mathcal{A},\mathcal{B},x_o^u,\mathfrak{X}_z^u\right) &= \Gamma\left(\mathcal{A},\mathcal{B},x_o^u,\mathfrak{Z}^u\right) = \frac{\frac{\partial f(\mathcal{A},x_o^u,\mathfrak{X}_z^u)}{\partial \mathcal{B},x_z^u}}{\frac{\partial g(\mathcal{B},\mathfrak{X}_z^u)}{\partial x_z^u}} = \frac{MP_1\left(\mathcal{A},x_o^u,\mathfrak{X}_z^u\right)}{MP_2\left(\mathcal{B},\mathfrak{X}_z^u\right)} \\ &= \Psi\left(\mathcal{A},\mathcal{B},x_o^1,\ldots,x_o^U,z\right) = \frac{\partial\mathcal{Y}\left(\mathcal{A},\mathcal{B},x_o^1,\ldots,x_o^U,z\right)}{\partial z} \end{aligned} \tag{10}$$

Thus, when allocative efficiency is achieved by the sector, the MACs are equalised across plants in the sector and this common (henceforth, sectoral) MAC is given by the Lagrange multiplier of problem (8) evaluated at the optimum, $\Psi\left(\mathcal{A},\mathcal{B},x_o^1,\ldots,x_o^U,z\right)$, which has the standard interpretation of the shadow price of the sector's emission cap.

The sectoral MAC schedule can be derived given a baseline level of sectoral emission $\bar{z}$ as

$$mac\left(\mathcal{A},\mathcal{B},x_o^1,\ldots,x_o^U,\theta\right) = \Psi\left(\mathcal{A},\mathcal{B},x_o^1,\ldots,x_o^U,\bar{z}-\theta\right)\ \forall\,\theta \in [0,\bar{z}]$$

For every level of sectoral abatement θ, the optimal allocation of the new sectoral emission defined as $z = \bar{z} - \theta$ among plants and the optimal allocation of the good output produced are obtained from (9).

3.6 Comparative statics of allocative efficiency.

A standard comparative static exercise can be conducted based on an application of the implicit function theorem on the first-order conditions of Problem (8).[10] The first-order conditions express the choice variables and the Lagrange multiplier of Problem (8) (the endogenous variables) as implicit functions of the parameters of the problem (the exogenous variables), namely, the distribution of fixed inputs in the sector, its emission cap, and the parameters of technological change. The comparative statics of Problem (8) study how, starting from an initial efficient allocation in the sector, the endogenous variables of concern will change due to changes in the

[10] We have done so elsewhere.

exogenous variables of concern. However, the intuition behind the results obtained through a standard comparative static exercise tend to be obscured by the heavy use of calculus in the proof. Hence, we present an alternative approach to arrive at these results. This approach rests on a useful and intuitive lemma presented below.

To sharpen the results of this analysis, throughout the remainder of this section based on a parametric approach, we will assume that functions f and g are, respectively, strictly concave and strictly convex.

Assumption 6 *Function f is strictly concave in x_o and x_z and function g is strictly convex in x_z.*

3.6.1 A useful lemma.

The proof of the lemma below rests on the arguments in the following remark that are based on the diminishing marginal productivity of the emission-causing input in the production of the good output and its increasing marginal productivity in the production of the bad output.

Remark 7 *Under Assumption 6, the $MAC(\mathcal{A}, \mathcal{B}, x_o, x_z) = MP_1(\mathcal{A}, x_o, x_z)/MP_2(\mathcal{B}, x_z)$ is increasing in x_z.*

Let $a = \{\langle x_o^u, x_z^u, y^u, z^u\rangle\}_{u=1}^U$ be an allocation where all plants are operating with technical efficiency (but the sector as a whole may or may not be operating with allocative efficiency). Then the MAC of each plant u at this allocation is given by (5) as the ratio of the marginal productivities of the emission-causing input in the production of the good and bad outputs for plant u. Now define the minimum and the maximum MACs obtained at allocation a, respectively, as

$$\begin{aligned} m^{min}(\mathcal{A}, \mathcal{B}, \mathbf{x}_o, \mathbf{x}_z) &:= \min\{MAC(\mathcal{A}, \mathcal{B}, x_o^1, x_z^1), \dots, MAC(\mathcal{A}, \mathcal{B}, x_o^U, x_z^U)\} \\ M^{max}(\mathcal{A}, \mathcal{B}, \mathbf{x}_o, \mathbf{x}_z) &:= \max\{MAC(\mathcal{A}, \mathcal{B}, x_o^1, x_z^1), \dots, MAC(\mathcal{A}, \mathcal{B}, x_o^U, x_z^U)\} \end{aligned} \tag{11}$$

where $\mathbf{x}_o = \langle x_o^1, \dots, x_o^U\rangle$ and $\mathbf{x}_z = \langle x_z^1, \dots, x_z^U\rangle$ are the distributions of fixed and emission generating inputs, respectively. It is clear that if allocation a is not allocatively efficient, then the MACs of plants are not equalised and $m^{min}(\mathcal{A}, \mathcal{B}, \mathbf{x}_o, \mathbf{x}_z) < M^{max}(\mathcal{A}, \mathcal{B}, \mathbf{x}_o, \mathbf{x}_z)$.

Lemma 8 simply states that the sectoral/common MAC that prevails when a sector operates with allocative efficiency with a given emission cap lies within the range of plant-level MACs observed at any technically efficient (but possibly allocatively inefficient) allocation in the sector with the same emission cap.

Lemma 8 *Suppose Assumption 6 holds. Let $\{\langle x_o^u, x_z^u, y^u, z^u\rangle\}_{u=1}^U$ be an allocation in a sector with technological parameters $\mathcal{A}$ and $\mathcal{B}$ and distribution of fixed inputs $\mathbf{x}_o = \langle x_o^1, \dots, x_o^U\rangle$, where all plants are operating with technical efficiency. Let $z = \sum_{u=1}^U z^u$ and $\mathbf{x}_z = \langle x_z^1, \dots, x_z^U\rangle$ denote the distribution of the emission generating inputs at the given allocation. Then the sectoral*

MAC obtained when the sector operates with allocative efficiency with emission cap z satisfies the following:

$$\Psi\left(\mathcal{A}, \mathcal{B}, \mathbf{x}_o, z\right) \in \left[m^{min}\left(\mathcal{A}, \mathcal{B}, \mathbf{x}_o, \mathbf{x}_z\right),\ M^{max}\left(\mathcal{A}, \mathcal{B}, \mathbf{x}_o, \mathbf{x}_z\right)\right].$$

Thus, as pointed out by Jung and Kruitilla (1996), firm-level variations in MAC due to firm-level heterogeneity provides a stimulus for emission trading.

In the following two sections we will assume that the status-quo is an allocation $a_{AE} = \{\langle x_o^u, \mathfrak{X}_z^u, \mathcal{Y}^u, \mathfrak{Z}^u\rangle\}_{u=1}^U$ where the sector operates with allocative efficiency given technological parameters $\mathcal{A}$ and $\mathcal{B}$, emission cap $z = \sum_{u=1}^U \mathfrak{Z}^u$, and the distribution of fixed inputs $\mathbf{x}_o = \langle x_o^1, \ldots, x_o^U\rangle$.[11] Starting from such a status-quo, we will study how the sectoral MAC changes as the sector moves to other allocatively efficient allocations due to changes in the distributions of the fixed inputs, the emission cap, or the technology.

3.6.2 The impact of a change in the distribution of fixed inputs on sectoral MAC.

Suppose, starting from the status-quo allocation a_{AE}, the levels of fixed inputs used by plant w increase with no change in usage of fixed inputs by the other plants: $\bar{x}_o^w > x_o^w$ and $\bar{x}_o^u = x_o^u$ for all $u = 1, \ldots, U$ with $u \neq w$. Now construct a technically efficient allocation a_{TE} with the following specifications: (i) All plants continue to use the same levels of the emission-causing input as in allocation a_{AE}, (ii) The fixed input usage by all plants other than w is at the same level as in a_{AE}, and (iii) the fixed input usage of plant w changes to its new level. Let $\bar{y}^w = f(\mathcal{A}, \bar{x}_o^w, \mathfrak{X}_z^w)$ and $\bar{z}^w = g(\mathcal{B}, \mathfrak{X}_z^w)$ be the new levels of good and bad outputs produced by plant w.

Since a_{AE} is allocatively efficient, the sectoral MAC at this allocation, given by $\Psi\left(\mathcal{A}, \mathcal{B}, \mathbf{x}_o, z\right)$, is also the MAC of all plants including plant w at this allocation. Now we make the following assumption that implies that the use of the fixed inputs such as capacity/capital in thermal power plants and the variable input (emission-causing input in our case) is complementary in production.

Assumption 9 *The marginal productivity of the emission-causing input in the production of the good output is increasing in the fixed inputs, i.e.,* $\frac{\partial MP_1(\mathcal{A}, x_o, x_z)}{\partial x_{o_i}} > 0$ *for* $i = 1, \ldots, N$.

Invoking Assumption 9, it follows that MP_1 will increase for plant w, whose usage of fixed inputs has increased, when we move from allocation a_{AE} to allocation a_{TE}. Hence, we have

$$\begin{aligned}\Psi\left(\mathcal{A}, \mathcal{B}, \mathbf{x}_o, z\right) &= MAC\left(\mathcal{A}, \mathcal{B}, x_o^w, \mathfrak{X}_z^w\right) = \frac{MP_1\left(\mathcal{A}, x_o^w, \mathfrak{X}_z^w\right)}{MP_2\left(\mathcal{B}, \mathfrak{X}_z^w\right)} \\ &< \frac{MP_1\left(\mathcal{A}, \bar{x}_o^w, \mathfrak{X}_z^w\right)}{MP_2\left(\bar{\mathcal{B}}, \mathfrak{X}_z^w\right)} = MAC\left(\mathcal{A}, \mathcal{B}, \bar{x}_o^w, \mathfrak{X}_z^w\right),\end{aligned}$$

[11] Thus, $\mathfrak{X}_z^u$, $\mathcal{Y}^u$, and $\mathfrak{Z}^u$ are as defined in (9) for all $u = 1, \ldots, U$.

while $\Psi(\mathcal{A},\mathcal{B},\mathbf{x}_o,z) = MAC(\mathcal{A},\mathcal{B},x_o^u,\mathfrak{X}_z^u) = \frac{MP_1(\mathcal{A},x_o^u,\mathfrak{X}_z^u)}{MP_2(\mathcal{B},\mathfrak{X}_z^u)}$ for all plants $u = 1,\ldots,U$ with $u \neq w$. The above shows that in moving from allocation a_{AE} to allocation a_{TE}, the MAC of plant w increases, while that of all other plants is unchanged. Therefore, recalling the definitions of functions m^{min} and M^{max} given in (11), we have for any $u \neq w$

$$\begin{aligned}\Psi(\mathcal{A},\mathcal{B},\mathbf{x}_o,z) = MAC(\mathcal{A},\mathcal{B},x_o^u,\mathfrak{X}_z^u) = m^{min}(\mathcal{A},\mathcal{B},\bar{\mathbf{x}}_o,\mathfrak{X}_z) &< M^{max}(\mathcal{A},\mathcal{B},\bar{\mathbf{x}}_o,\mathfrak{X}_z) \\ &= MAC(\mathcal{A},\mathcal{B},\bar{x}_o^w,\mathfrak{X}_z^w) \quad (12)\end{aligned}$$

where $\bar{\mathbf{x}}_o = \langle x_o^1,\ldots,x_o^{w-1},\bar{x}_o^w,x_o^{w+1},\ldots,x_o^U\rangle$. An application of Lemma 8 implies that the sectoral MAC obtained when the distribution of fixed inputs is $\bar{\mathbf{x}}_o$ lies in the range of plant-level MACs obtained at the technically efficient allocation a_{TE}:
$\Psi(\mathcal{A},\mathcal{B},\bar{\mathbf{x}}_o,z) \in [m^{min}(\mathcal{A},\mathcal{B},\bar{\mathbf{x}}_o,\mathfrak{X}_z),\ M^{max}(\mathcal{A},\mathcal{B},\bar{\mathbf{x}}_o,\mathfrak{X}_z)]$. Combined with (12) this implies

$$\Psi(\mathcal{A},\mathcal{B},\mathbf{x}_o,z) \leq \Psi(\mathcal{A},\mathcal{B},\bar{\mathbf{x}}_o,z).$$

Thus, we have the following comparative static result:

Proposition 10 *Suppose Assumptions 6 and 9 hold. Ceteris-paribus, increases in the usage of fixed inputs by plant w have a non-decreasing impact on the sectoral MAC.*

3.6.3 The impact of a change in the emission cap on sectoral MAC.

Suppose, starting from the status-quo allocation a_{AE}, the emission cap increases from $z = \sum_{u=1}^{U}\mathfrak{Z}^u$ to $\bar{z}$, where $\mathfrak{Z}^u$ is the minimum emission that can be generated when each plant u employs $\mathfrak{X}_z^u$ amount of emission-causing input. Let $\Delta z = \bar{z} - z > 0$. Now it is possible to construct a technically efficient allocation a_{TE} where

- The fixed inputs of all plants are employed at the same level as in a_{AE}.
- The emission-causing input usage of each plant is increased by an amount $\Delta x_z > 0$ so that the sector can produce the new higher level of sectoral emission $\bar{z}$, *i.e.,* Δx_z is such that it solves
$$\sum_{u=1}^{U} g(\mathcal{B},\mathfrak{X}_z^u + \Delta x_z) = \bar{z}.$$

Let $\bar{x}_z^u = \mathfrak{X}_z^u + \Delta x_z$ for all $u = 1,\ldots,U$. Then $a_{TE} = \left\{\langle x_o^u,\bar{x}_z^u,\bar{y}^u,\bar{z}^u\rangle\right\}_{u=1}^{U}$ is a technically efficient allocation, where $\bar{y}^u = f(\mathcal{A},x_o^w,\bar{x}_z^u)$ and $\bar{z}^u = g(\mathcal{B},\bar{x}_z^u)$ for all $u = 1,\ldots,U$.

Since $\bar{x}_z^u > \mathfrak{X}_z^u$ for all $u = 1,\ldots,U$, it follows from the strict concavity of function f and the strict convexity of function g (which imply diminishing and increasing marginal productivity of x_z in the production of the good and bad outputs, respectively) that

$$\begin{aligned}\Psi(\mathcal{A},\mathcal{B},\mathbf{x}_o,z) &= MAC(\mathcal{A},\mathcal{B},x_o^u,\mathfrak{X}_z^u) = \frac{MP_1(\mathcal{A},x_o^u,\mathfrak{X}_z^u)}{MP_2(\mathcal{B},\mathfrak{X}_z^u)} \\ &> \frac{MP_1(\mathcal{A},x_o^u,\bar{x}_z^u)}{MP_2(\bar{\mathcal{B}},\bar{x}_z^u)} = MAC(\mathcal{A},\mathcal{B},x_o^u,\bar{x}_z^u)\ \forall\ u = 1,\ldots,U\end{aligned}$$

Therefore, recalling the definitions of functions m^{min} and M^{max} given in (11), we have

$$\Psi\left(\mathcal{A}, \mathcal{B}, \mathbf{x}_o, z\right) > M^{max}\left(\mathcal{A}, \mathcal{B}, \mathbf{x}_o, \bar{\mathbf{x}}_z\right) \geq m^{min}\left(\mathcal{A}, \mathcal{B}, \mathbf{x}_o, \bar{\mathbf{x}}_z\right), \tag{13}$$

where $\bar{\mathbf{x}}_z = \langle \bar{x}_z^1, \ldots, \bar{x}_z^U \rangle$. But Lemma 8 implies that the sectoral MAC associated with the new emission cap lies in the range of MACs at allocation a_{TE} where the new cap also holds:

$$\Psi\left(\mathcal{A}, \mathcal{B}, \mathbf{x}_o, \bar{z}\right) \in [m^{min}\left(\mathcal{A}, \mathcal{B}, \mathbf{x}_o, \bar{\mathbf{x}}_z\right),\ M^{max}\left(\mathcal{A}, \mathcal{B}, \mathbf{x}_o, \bar{\mathbf{x}}_z\right)].$$

Combined with (13), this means that

$$\Psi\left(\mathcal{A}, \mathcal{B}, \mathbf{x}_o, z\right) > \Psi\left(\mathcal{A}, \mathcal{B}, \mathbf{x}_o, \bar{z}\right).$$

We we have the following comparative static result for the sectoral MAC.

Proposition 11 (The law of cap induced diminishing sectoral MAC): *Suppose Assumption 6 holds. Ceteris-paribus, an increase in the sectoral emission cap has a decreasing impact on the sectoral MAC.*

3.6.4 The impact of technological change on sectoral MAC.

In the analysis presented below, the impact of a technological change from $T(\mathcal{A}, \mathcal{B})$ to $T(\bar{\mathcal{A}}, \bar{\mathcal{B}})$, where $\bar{\mathcal{A}} > \mathcal{A}$ and $\bar{\mathcal{B}} > \mathcal{B}$, on the sectoral MAC is studied in two stages when the emission cap and the distribution of fixed inputs are held fixed at z and $\mathbf{x}_o$, respectively: *Stage 1* covers the impact on the sectoral MAC due to a change in sub-technology T_2 only, so that the overall technology changes from $T(\mathcal{A}, \mathcal{B})$ to $T(\mathcal{A}, \bar{\mathcal{B}})$. *Stage 2* captures the additional impact on the sectoral MAC when sub-technology T_1 also changes, *i.e.,* the overall technology changes from $T(\mathcal{A}, \bar{\mathcal{B}})$ to $T(\bar{\mathcal{A}}, \bar{\mathcal{B}})$.

Stage 1: Change in sectoral MAC due to technological change in T_2.

(i) Starting from the status-quo allocatively efficient allocation $a_{AE} = \{\langle x_o^u, \mathfrak{X}_z^u, \mathcal{Y}^u, \mathfrak{Z}^u \rangle\}_{u=1}^U$ where sectoral emission cap is $z = \sum_{u=1}^U \mathfrak{Z}^u$, we first construct a technically efficient allocation $a_{TE} = \{\langle x_o^u, \mathfrak{X}_z^u, \mathcal{Y}^u, \bar{z}^u \rangle\}_{u=1}^U$, where each plant operates with inputs that it was using in the allocation a_{AE}, but when sub-technology T_2 changes from $T_2(\mathcal{B})$ to $T_2(\bar{\mathcal{B}})$. With no change in the usage of emission-causing input, this implies a new level of emission generated by each plant given by $\bar{z}^u = g\left(\bar{\mathcal{B}}, \mathfrak{X}_z^u\right)$ for all $u = 1 \ldots, U$. The sectoral emission is thus $\bar{z} := \sum_{u=1} \bar{z}^u$. In particular, if the technological change is clean, then $\bar{z} < z$, while if the technological change is dirty then $\bar{z} > z$. Since input usage and sub-technology T_1 have not changed, there is no change in the good output production by the plants in the move from allocation a_{AE} to allocation a_{TE}.

(ii) An application of Lemma 8 implies that the sectoral MAC at allocation $\bar{a}_{AE}$ that is achieved when the sector operates with allocative efficiency with emission cap $\bar{z}$ and technology $T(\mathcal{A},\bar{\mathcal{B}})$ lies in the range of plant-level MACs at allocation a_{TE}:

$$\Psi\left(\mathcal{A},\bar{\mathcal{B}},\mathbf{x}_o,\bar{z}\right)\;\in\;\left[m^{min}\left(\mathcal{A},\bar{\mathcal{B}},\mathbf{x}_o,\mathfrak{X}_z\right),\;M^{max}\left(\mathcal{A},\bar{\mathcal{B}},\mathbf{x}_o,\mathfrak{X}_z\right)\right], \tag{14}$$

where $\mathfrak{X}_z=\langle\mathfrak{X}_z^1,\ldots,\mathfrak{X}_z^U\rangle$.

Defining the change in sectoral MAC due to the change in sub-technology T_2 from $T_2(\mathcal{B})$ to $T_2(\bar{\mathcal{B}})$ and the change in the sectoral emission cap from z to $\bar{z}$ as

$$\Delta\Psi_{\mathcal{B}}^{(1)}:=\Psi\left(\mathcal{A},\bar{\mathcal{B}},\mathbf{x}_o,\bar{z}\right)-\Psi\left(\mathcal{A},\mathcal{B},\mathbf{x}_o,z\right).$$

Then, depending upon whether the sectoral MAC at the status-quo allocation a_{AE} lies in or to the left or right of the interval defined in (14), we have

$$\begin{aligned}
\Psi\left(\mathcal{A},\mathcal{B},\mathbf{x}_o,z\right)\;&\leq\;m^{min}\left(\mathcal{A},\bar{\mathcal{B}},\mathbf{x}_o,\mathfrak{X}_z\right)\implies\Psi\left(\mathcal{A},\bar{\mathcal{B}},\mathbf{x}_o,\bar{z}\right)\;\geq\;\Psi\left(\mathcal{A},\mathcal{B},\mathbf{x}_o,z\right)\\
&\implies\Delta\Psi_{\mathcal{B}}^{(1)}\;\geq\;0\\
\Psi\left(\mathcal{A},\mathcal{B},\mathbf{x}_o,z\right)\;&\geq\;M^{max}\left(\mathcal{A},\bar{\mathcal{B}},\mathbf{x}_o,\mathfrak{X}_z\right)\implies\Psi\left(\mathcal{A},\bar{\mathcal{B}},\mathbf{x}_o,\bar{z}\right)\;\leq\;\Psi\left(\mathcal{A},\mathcal{B},\mathbf{x}_o,z\right)\\
&\implies\Delta\Psi_{\mathcal{B}}^{(1)}\;\leq\;0.\\
\Psi\left(\mathcal{A},\mathcal{B},\mathbf{x}_o,z\right)\;&\in\;\left[m^{min}\left(\mathcal{A},\bar{\mathcal{B}},\mathbf{x}_o,\mathfrak{X}_z\right),\;M^{max}\left(\mathcal{A},\bar{\mathcal{B}},\mathbf{x}_o,\mathfrak{X}_z\right)\right]\\
&\implies\text{the sign of }\Delta\Psi_{\mathcal{B}}^{(1)}\text{ is ambiguous.}
\end{aligned} \tag{15}$$

(iii) We now move from allocation $\bar{a}_{AE}$ achieved in Step (ii) with cap $\bar{z}$ to allocation $\bar{\bar{a}}_{AE}$ that is achieved when the sector operates with allocative efficiency with overall technology $T(\mathcal{A},\bar{\mathcal{B}})$ and the original emission cap z: $\bar{\bar{a}}_{AE}=\left\{\left\langle x_o^u(t),\bar{\bar{\mathfrak{X}}}_z^u,\bar{\bar{\mathcal{Y}}}^u,\bar{\bar{\mathfrak{Z}}}^u\right\rangle\right\}_{u=1}^U$, where $\bar{\bar{\mathfrak{X}}}_z^u=\mathfrak{X}_z^u\left(\mathcal{A},\bar{\mathcal{B}},\mathbf{x}_o,z\right)$, $\bar{\bar{\mathcal{Y}}}^u=\mathcal{Y}^u\left(\mathcal{A},\bar{\mathcal{B}},\mathbf{x}_o,z\right)$, and $\bar{\bar{\mathfrak{Z}}}^u=\mathfrak{Z}^u\left(\mathcal{A},\bar{\mathcal{B}},\mathbf{x}_o,z\right)$ for all $u=1,\ldots,U$ and $\sum_{u=1}^U\bar{\bar{\mathfrak{Z}}}^u=z$.

Define the change in sectoral MAC when the sectoral emission cap reverts to z from $\bar{z}$ with technology $T(\mathcal{A},\bar{\mathcal{B}})$ as

$$\Delta\Psi_{\mathcal{B}}^{(2)}:=\Psi\left(\mathcal{A},\bar{\mathcal{B}},\mathbf{x}_o,z\right)-\Psi\left(\mathcal{A},\bar{\mathcal{B}},\mathbf{x}_o,\bar{z}\right).$$

Thus, the change in the sectoral MAC at the end of *Stage 1* can be decomposed into (a) the change in the sectoral MAC due to a change in the sub-technology T_2 to $T(\mathcal{B})$ and the emission cap to $\bar{z}$ reflecting the fact that the original levels of the emission-causing input used by the plants can no longer sustain their original levels of the emission and (b) the change in sectoral MAC due to reverting back to the original cap with the new sub-technology T_2.

$$\begin{aligned}
\Delta\Psi_{\mathcal{B}}\;&:=\;\Psi\left(\mathcal{A},\bar{\mathcal{B}},\mathbf{x}_o,z\right)-\Psi\left(\mathcal{A},\mathcal{B},\mathbf{x}_o,z\right)\\
&=\;\left[\Psi\left(\mathcal{A},\bar{\mathcal{B}},\mathbf{x}_o,\bar{z}\right)-\Psi\left(\mathcal{A},\mathcal{B},\mathbf{x}_o,z\right)\right]\;+\;\left[\Psi\left(\mathcal{A},\bar{\mathcal{B}},\mathbf{x}_o,z\right)-\Psi\left(\mathcal{A},\bar{\mathcal{B}},\mathbf{x}_o,\bar{z}\right)\right]\\
&=\;\Delta\Psi_{\mathcal{B}}^{(1)}\;+\;\Delta\Psi_{\mathcal{B}}^{(2)}
\end{aligned} \tag{16}$$

Proposition 12 *Suppose Assumption 6 holds. From (i) to (iii) of Stage 1 it follows that*

(a) $MP_2(\bar{\mathcal{B}}, \mathfrak{X}_z^u) < MP_2(\mathcal{B}, \mathfrak{X}_z^u)\ \forall\ u = 1, \ldots, U \implies \Delta\Psi_{\mathcal{B}}^{(1)} > 0$

(b) $MP_2(\bar{\mathcal{B}}, \mathfrak{X}_z^u) > MP_2(\mathcal{B}, \mathfrak{X}_z^u)\ \forall\ u = 1, \ldots, U \implies \Delta\Psi_{\mathcal{B}}^{(1)} < 0.$

(c) if $MP_2(\bar{\mathcal{B}}, \mathfrak{X}_z^u) < MP_2(\mathcal{B}, \mathfrak{X}_z^u)$ *and* $MP_2(\bar{\mathcal{B}}, \mathfrak{X}_z^w) > MP_2(\mathcal{B}, \mathfrak{X}_z^w)$ *for some* u, w *such that* $u \neq w$ *then the sign of* $\Delta\Psi_{\mathcal{B}}^{(1)}$ *is ambiguous.*

(d) If the technological change is dirty then $z < \bar{z}$ *and* $\Delta\Psi_{\mathcal{B}}^{(2)} > 0$

(e) If the technological change is clean then $z > \bar{z}$ *and* $\Delta\Psi_{\mathcal{B}}^{(2)} < 0$

Parts (a) and (b) of Proposition 12 state that if MP_2 at the original level of input usage falls (respectively, rises) for all plants due to the technological change in sub-technology T_2, then the resulting change in the sectoral MAC with cap $\bar{z}$, given by $\Delta\Psi_{\mathcal{B}}^{(1)}$, will be positive (respectively, negative). On the other hand, if MP_2 rises for some plants and falls for others, then the sign of $\Delta\Psi_{\mathcal{B}}^{(1)}$ is ambiguous. Part (a) follows because $MP_2(\bar{\mathcal{B}}, \mathfrak{X}_z^u) < MP_2(\mathcal{B}, \mathfrak{X}_z^u)\ \forall\ u = 1, \ldots, U$ implies

$$\begin{aligned}
\Psi(\mathcal{A}, \mathcal{B}, \mathbf{x}_o, z) &= MAC(\mathcal{A}, \mathcal{B}, x_o^u, \mathfrak{X}_z^u) = \frac{MP_1(\mathcal{A}, x_o^u, \mathfrak{X}_z^u)}{MP_2(\mathcal{B}, \mathfrak{X}_z^u)} \\
&< \frac{MP_1(\mathcal{A}, x_o^u, \mathfrak{X}_z^u)}{MP_2(\bar{\mathcal{B}}, \mathfrak{X}_z^u)} = MAC(\mathcal{A}, \bar{\mathcal{B}}, x_o^u, \mathfrak{X}_z^u) \quad \forall\ u = 1, \ldots, U \\
&\implies \Psi(\mathcal{A}, \mathcal{B}, \mathbf{x}_o, z) < m^{min}(\mathcal{A}, \bar{\mathcal{B}}, \mathbf{x}_o, \mathfrak{X}_z),
\end{aligned}$$

where we recall that $m^{min}(\mathcal{A}, \bar{\mathcal{B}}, \mathbf{x}_o, \mathfrak{X}_z)$ is the minimum of the range of MACs of plants prevailing at allocation a_{TE}. Hence, (15) implies Part (a). Similarly we can prove Parts (b) and (c) of Proposition 12.[12] Parts (d) and (e) of Proposition 12 follow in a straightforward manner from the law of cap induced diminishing sectoral MAC (see Remark 11), which says that changes in the sectoral MAC and the sectoral emission cap are inversely related. Hence, when the sector reverts back to the original level of the emission cap z from $\bar{z}$, then the sectoral MAC will fall if the technological change in T_2 is clean (so that $\bar{z} < z$) and will rise it is dirty (so that $\bar{z} > z$).

The remark below presents the conclusions that can be drawn from Proposition 12. It comprehensively lists the various ways in which a change in sub-technology T_2 can effect the sectoral MAC.

Remark 13 *Suppose Assumption 6 holds. Proposition 12 implies that*

(a) If the technological change in T_2 *is decreasingly clean for all* $u = 1, \ldots, U$ *at allocation* a_{TE} *then (b) and (e) of Proposition 12 hold and* $\Delta\Psi_{\mathcal{B}} < 0$ *and hence the sectoral MAC will locally fall.*

[12] For example, Part (b) can be proved by by showing that

$$MP_2(\bar{\mathcal{B}}, \mathfrak{X}_z^u) > MP_2(\mathcal{B}, \mathfrak{X}_z^u)\ \forall\ u = 1, \ldots, U \implies \Psi(\mathcal{A}, \mathcal{B}, \mathbf{x}_o, z) > M^{max}(\mathcal{A}, \bar{\mathcal{B}}, \mathbf{x}_o, \mathfrak{X}_z)$$

and employing (15).

(b) *If the technological change in T_2 is decreasingly dirty for all $u = 1, \ldots, U$ at allocation a_{TE} then (a) and (d) of Proposition 12 hold and $\Delta\Psi_{\mathcal{B}} > 0$ and hence the sectoral MAC will locally rise.*

(c) *The sign of $\Delta\Psi_{\mathcal{B}}$ and hence the sign of the change in the sectoral MAC is locally ambiguous if the technological change in T_2 is*

i. *such that (c) of Proposition 12 holds locally.*

ii. *increasingly clean for all $u = 1, \ldots, U$ at allocation a_{TE} as then (a) and (e) of Proposition 12 will locally hold.*

iii. *increasingly dirty for all $u = 1, \ldots, U$ at allocation a_{TE} as then (b) and (d) of Proposition 12 will locally hold.*

Stage 2: Change in sectoral MAC due to technological change in T_1.

(i) We move from the allocation $\bar{\bar{a}}_{AE}$ achieved at the end of *Stage 1*, where the sector operates with allocative efficiency with the new sub-technology $T_2(\bar{\mathcal{B}})$ and emission cap z, to the technically efficient allocation $\bar{\bar{a}}_{TE} = \left\{\left\langle x_o^u, \bar{\bar{\mathfrak{X}}}_z^u, \bar{\bar{y}}^u, \bar{\bar{\mathfrak{z}}}^u \right\rangle\right\}_{u=1}^{U}$, where each plant operates with inputs that it was using in the efficient allocation $\bar{\bar{a}}_{AE}$ but when sub-technology T_1 also changes from $T_1(\mathcal{A})$ to $T_1(\bar{\mathcal{A}})$.

This change in the sub-technology T_1 will, in general, lead to a change in the level of the good output produced by each plant given by $\bar{\bar{y}}^u = f\left(\bar{\mathcal{A}}, x_o^u, \bar{\bar{\mathfrak{X}}}_z^u\right)$ for all $u = 1 \ldots, U$. Since input usage and sub-technology T_2 have not changed in the move from allocation $\bar{\bar{a}}_{AE}$ to allocation $\bar{\bar{a}}_{TE}$, there is no change in emission generation by the plants in this move.

(ii) We next move from the technically efficient allocation $\bar{\bar{a}}_{TE}$ under the new technology $T(\bar{\mathcal{A}}, \bar{\mathcal{B}})$ to an allocatively efficient allocation, denoted by $\hat{a}_{AE}$, under this technology. It follows from Lemma 8 that the sectoral MAC under allocation $\hat{a}_{AE}$ lies in the range of plant-level MACs at allocation $\bar{\bar{a}}_{TE}$:

$$\Psi\left(\bar{\mathcal{A}}, \bar{\mathcal{B}}, \mathbf{x}_o, z\right) \in \left[m^{min}\left(\bar{\mathcal{A}}, \bar{\mathcal{B}}, \mathbf{x}_o, \bar{\bar{\mathfrak{X}}}_z\right),\ M^{max}\left(\bar{\mathcal{A}}, \bar{\mathcal{B}}, \mathbf{x}_o, \bar{\bar{\mathfrak{X}}}_z\right)\right], \tag{17}$$

where $\bar{\bar{\mathfrak{X}}}_z = \langle \bar{\bar{\mathfrak{X}}}_z^1, \ldots, \bar{\bar{\mathfrak{X}}}_z^U \rangle$. Defining the change in sectoral MAC due to change in sub-technology T_1 from $T_1(\mathcal{A})$ to $T_1(\bar{\mathcal{A}})$ as

$$\Delta\Psi_{\mathcal{A}} := \Psi\left(\bar{\mathcal{A}}, \bar{\mathcal{B}}, \mathbf{x}_o, z\right) - \Psi\left(\mathcal{A}, \bar{\mathcal{B}}, \mathbf{x}_o, z\right),$$

we have the following depending upon whether the sectoral MAC at allocation $\bar{\bar{a}}_{AE}$ lies in or to the left or to the right of the interval defined in (17):

$$\begin{aligned}
\Psi\left(\mathcal{A},\bar{\mathcal{B}},\mathbf{x}_o,z\right) &\leq m^{min}\left(\bar{\mathcal{A}},\bar{\mathcal{B}},\mathbf{x}_o,\bar{\bar{\mathfrak{X}}}_z\right) \implies \Psi\left(\bar{\mathcal{A}},\bar{\mathcal{B}},\mathbf{x}_o,z\right) \geq \Psi\left(\mathcal{A},\bar{\mathcal{B}},\mathbf{x}_o,z\right) \\
&\implies \Delta\Psi_{\mathcal{A}} \geq 0 \quad \text{and} \\
\Psi\left(\mathcal{A},\bar{\mathcal{B}},\mathbf{x}_o,z\right) &\geq M^{max}\left(\bar{\mathcal{A}},\bar{\mathcal{B}},\mathbf{x}_o,\bar{\bar{\mathfrak{X}}}_z\right) \implies \Psi\left(\bar{\mathcal{A}},\bar{\mathcal{B}},\mathbf{x}_o,z\right) \leq \Psi\left(\mathcal{A},\bar{\mathcal{B}},\mathbf{x}_o,z\right) \\
&\implies \Delta\Psi_{\mathcal{A}} \leq 0 \\
\Psi\left(\bar{\mathcal{A}},\bar{\mathcal{B}},\mathbf{x}_o,z\right) &\in \left[m^{min}\left(\bar{\mathcal{A}},\bar{\mathcal{B}},\mathbf{x}_o,\bar{\bar{\mathfrak{X}}}_z\right),\ M^{max}\left(\bar{\mathcal{A}},\bar{\mathcal{B}},\mathbf{x}_o,\bar{\bar{\mathfrak{X}}}_z\right)\right] \\
&\implies \text{the sign of } \Delta\Psi_{\mathcal{A}} \text{ is ambiguous.}
\end{aligned} \tag{18}$$

The proposition below is obtained in a manner similar to Parts (a) to (c) of Proposition 12. It provides some specific situations when the sectoral MAC at allocation $\bar{\bar{a}}_{AE}$ can be expected to lie to the left or the right of the interval of plant level MACs at the technically allocation $\bar{\bar{a}}_{TE}$ identified in (17). For example, Part (a) of the proposition says that if MP_1 of all plants increases due to the change in sub-technology T_1 with all plants continuing to use the levels of inputs they were using at the end of *Stage 1*, then the sectoral MAC will increase.

Proposition 14 *Suppose Assumption 6 holds. From (i) to (iii) of Stage 2 it follows that*

(a) $MP_1\left(\bar{\mathcal{A}},x_o^u,\bar{\bar{\mathfrak{X}}}_z^u\right) > MP_1\left(\mathcal{A},x_o^u,\bar{\bar{\mathfrak{X}}}_z^u\right)\ \forall\ u=1,\ldots,U \implies \Delta\Psi_{\mathcal{A}} \geq 0$

(b) $MP_1\left(\bar{\mathcal{A}},x_o^u,\bar{\bar{\mathfrak{X}}}_z^u\right) < MP_1\left(\mathcal{A},x_o^u,\bar{\bar{\mathfrak{X}}}_z^u\right)\ \forall\ u=1,\ldots,U \implies \Delta\Psi_{\mathcal{A}} \leq 0$

(c) $MP_1\left(\bar{\mathcal{A}},x_o^u,\bar{\bar{\mathfrak{X}}}_z^u\right) > MP_1\left(\mathcal{A},x_o^u,\bar{\bar{\mathfrak{X}}}_z^u\right)$ *and* $MP_1\left(\bar{\mathcal{A}},x_o^w,\bar{\bar{\mathfrak{X}}}_z^w\right) < MP_1\left(\mathcal{A},x_o^w,\bar{\bar{\mathfrak{X}}}_z^w\right)$ *for some* u, w *such that* $u \neq w \implies$ *the sign of* $\Delta\Psi_{\mathcal{A}}$ *is ambiguous.*

We conclude from *Stages 1 and 2* above that the total change in the sectoral MAC due to technological changes in sub-technologies T_1 and T_2, denoted by $\Delta\Psi_{\mathcal{T}}$, can be decomposed into the total change in the sectoral MAC in *Stage 1*, where sub-technology T_2 alone was allowed to change, and the change in sectoral MAC in *Stage 2*, where sub-technology T_1 is also allowed to change:

$$\begin{aligned}
\Delta\Psi_{\mathcal{T}} &:= \Psi\left(\bar{\mathcal{A}},\bar{\mathcal{B}},\mathbf{x}_o,z\right) - \Psi\left(\mathcal{A},\mathcal{B},\mathbf{x}_o,z\right) \\
&= \left[\Psi\left(\bar{\mathcal{A}},\bar{\mathcal{B}},\mathbf{x}_o,z\right) - \Psi\left(\mathcal{A},\bar{\mathcal{B}},\mathbf{x}_o,z\right)\right] + \left[\Psi\left(\mathcal{A},\bar{\mathcal{B}},\mathbf{x}_o,z\right) - \Psi\left(\mathcal{A},\mathcal{B},\mathbf{x}_o,z\right)\right] \\
&= \Delta\Psi_{\mathcal{A}} + \Delta\Psi_{\mathcal{B}}
\end{aligned} \tag{19}$$

4 Comparative statics of sectoral MAC: A non-parametric analysis.

Our empirical work is based on a non-parametric DEA approach. In fact the theory in this paper, which was developed in the previous sections using a parametric analysis, was motivated by the results and our understanding of these results from our empirical non-parametric work. To evaluate whether the performance of a sector is in keeping with the climate change objectives of the government requires specification of a flexible production model that can capture any scenario of performance of the production units and the sector based on the data fed to it. We believe that this is harder to achieve in our context using the parametric approach because it is harder to come up with flexible enough functional forms for functions f and g that stand ready to capture, depending on the data, any scenario of (locally) increasingly or decreasingly clean or dirty technological change in T_2 and any scenario of (locally) progressive or regressive technological change in T_1.

Our non-parametric analysis captures the nature of the technological change and its impact on the sectoral MAC in a straightforward manner. In this section, we develop a non-parametric model of production for this purpose and define the concept of allocative efficiency under this approach. We provide a methodology to compute the plant-level and sector level MAC and show how changes in the sectoral MAC over time can be decomposed into a component that can be attributed purely to change in the distribution of fixed inputs in the sector, a component due to change in the emission cap of the sector, and a component due to a technological change.

4.1 A non-parametric specification of a by-production technology.

Consider a $(N+1) \times U$-dimensional data matrix of inputs used by the U firms/plants in the sector, denoted by X, which is decomposed as

$$X = \begin{bmatrix} X_o & X_z \end{bmatrix}$$

where X_o is the $N \times U$-dimensional data matrix of the N non-emission generating inputs and X_z is the $1 \times U$-dimensional data matrix of the emission-generating inputs. Y and Z are, respectively, the $1 \times U$-dimensional data matrices of good output and the bad output, emission. We will assume that data matrices X_z and Z data matrices are such that for no plant u we have $X_z(u) > 0$ and $Z(u) = 0$, that is, any plant in the dataset that uses positive amount of the emission-causing input produces positive amount of emission.

Following MRL, the DEA non-parametric representation of a by-production technology under the assumptions of convexity and non-increasing returns to scale is given by[13]

$$\begin{aligned}
T(\hat{t}, t) &= T_1(\hat{t}) \cap T_2(t), \text{ where} \\
T_1(\hat{t}) &= \Big\{ \langle x_o, x_z, y, z \rangle \in \mathbf{R}_+^{N+3} \mid \lambda^\top X_o^{\hat{t}} \leq x_o,\ \lambda^\top Y^{\hat{t}} \geq y,\ \lambda^\top X_z^{\hat{t}} \leq x_z \\
&\qquad \lambda \geq 0_U,\ \sum_{u=1}^{U} \lambda_u \leq 1 \Big\} \text{ and} \\
T_2(t) &= \Big\{ \langle x_o, x_z, y, z \rangle \in \mathbf{R}_+^{N+3} \mid \mu^\top X_z^t \geq x_z,\ \mu^\top Z^t \leq z,\ \sum_{u=1}^{U} \mu_u \leq 1,\ \mu \geq 0_U \Big\}. \quad (20)
\end{aligned}$$

[13] 0_U stands for a U-dimensional column vector with all elements as zero. Similarly, we can define 1_U, *etc.*

This non-parametric specification allows technological change by allowing technology to be time dependent. In particular, it allows the two sub-technologies T_1 and T_2 to be of different vintages, $\hat{t}$ and t, respectively, to capture independent technological change in these two sub-technologies. Thus, while T_1 is determined by data matrices $X^{\hat{t}}$, $Y^{\hat{t}}$, and $Z^{\hat{t}}$ on decision making units prevailing at time point $\hat{t}$, T_2 is determined by data matrices X^t and Z^t on decision making units prevailing at time point t.

4.2 Non-parametric approach for computing MAC of a plant and its decomposition.

Both dual and primal linear programming approaches will be used to define and characterise the MAC of a plant under the non-parametric approach. The non-smooth nature of the frontiers of the technologies constructed using the non-parametric approach imply that the MAC of a plant may not be unique. The primal linear programming approach is helpful in our case for directly identifying the lower and upper bounds of the MAC.

4.2.1 Non-parametric computation of MAC of power plants at the status-quo: A dual approach.

Analogous to Problem (4) in the parametric case, the problem below maximises production of the good output by plant u subject to the levels of the non emission-generating inputs given by the vector x_o^u and the level of the emission, z^u. The non-parametric formulation of $T(\hat{t}, t)$ given in (20) is employed in this problem.

$$\max_{x_z^u, y^u, \lambda^u, \mu^u} \{y^u \mid \langle x_o^u, x_z^u, y^u, z^u \rangle \in T(\hat{t}, t)\} \tag{21}$$

Analogous to the conclusions drawn from (5) in Section 3.2, the shadow price of the emission constraint of problem (21) has the interpretation of the MAC of plant u. Hence, the mathematical dual of the primal linear programming problem (21) will yield to us the relevant shadow price (MAC) of emission of plant u. To obtain this, let us rewrite this primal problem using matrix notation as follows:[14]

$$\max_{\delta} \left\{f^\top \delta \mid A\delta \leq b\right\}$$

[14]We are ignoring the time superscript with a view to simplify the exposition.

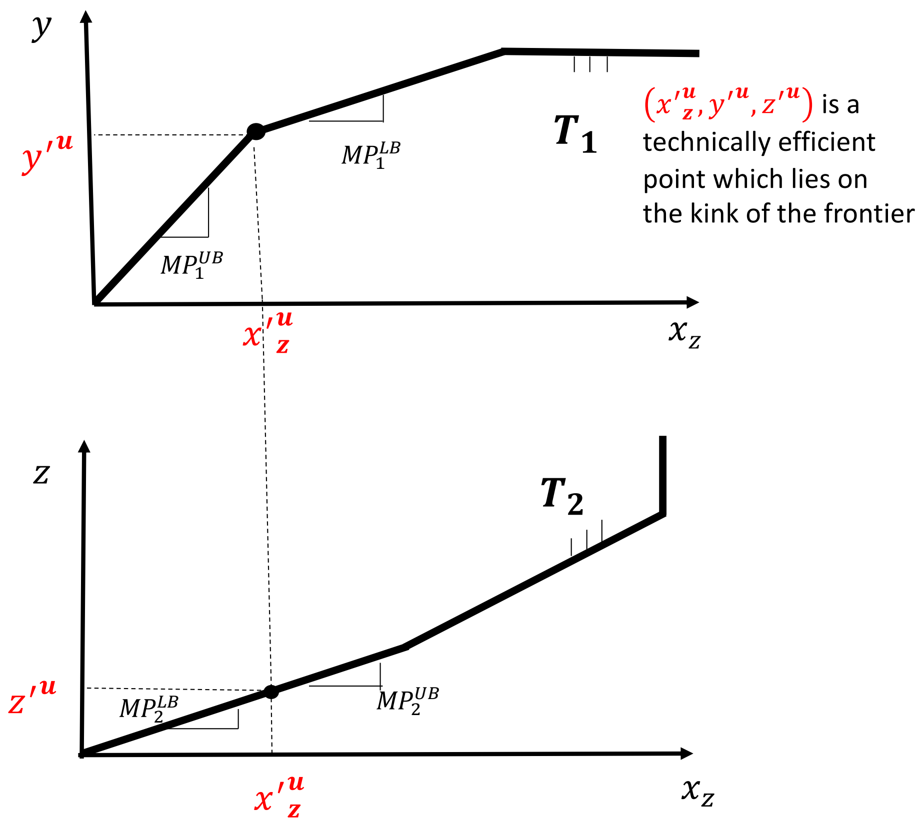
where

$$A = \left\{ \begin{array}{cccc} X_o & 0_{N_1 \times U} & 0_{N_1 \times N_2} & 0_{N_1} \\ X_z & 0_{N_2 \times U} & -I_{N_2 \times N_2} & 0_{N_2} \\ -Y & 0_U^\top & 0_{N_2}^\top & 1 \\ 0_{N_2 \times U} & -X_z & I_{N_2 \times N_2} & 0_{N_2} \\ 0_U^\top & Z & 0_{N_2}^\top & 0 \\ 1_U^\top & 0_U^\top & 0_{N_2}^\top & 0 \\ 0_U^\top & 1_U^\top & 0_{N_2}^\top & 0 \end{array} \right\}, \quad f = \left\{ \begin{array}{c} 0_U \\ 0_U \\ 0_{N_2} \\ 1 \end{array} \right\}, \quad b = \left\{ \begin{array}{c} \hat{x}_o^u \\ 0_{N_2} \\ 0 \\ 0_{N_2} \\ \hat{z}^u \\ 1 \\ 1 \end{array} \right\}, \quad \text{and} \quad \delta^u = \left\{ \begin{array}{c} \lambda^u \\ \mu^u \\ x_z \\ y \end{array} \right\}.$$

The mathematical dual of this primal problem is given by

$$\min_{\rho^u} \left\{ b^\top \rho^u \mid A^\top \rho^u \geq f \right\} \tag{22}$$

Solving problem (22) given data on plants we obtain ρ^u, which is the vector of shadow prices of all constraints of the primal problem (21) including the shadow price of emission or the MAC for plant u. For example, in our empirical application with two non-emission-causing inputs and one emission-causing input, the emission constraint is the fifth constraint of primal problem (21). Thus, the fifth element of the computed vector ρ^u, denoted by γ^u, will give us the estimated shadow price of emission (MAC) of firm u.



Non-parametric representation of by-production technology

Figure 7

Note that the value of MAC estimated by using the above non-parametric DEA approach may not be unique. This is because the technological frontier estimated using this approach is piece-wise linear and not smooth. This can be seen from Figure 7 which represents a by-production technology drawn holding the levels of non emission-causing inputs fixed. The upper panel of the figure shows sub-technology T_1 in the space of y and x_z and the lower panel of the figure shows T_2 in the space of z and x_z. We obtain a unique solution for (22) only when plant u operates on the flat portion of the frontier of both T_1 and T_2. When it operates on a kink of the technological frontier such that its production vector is $v'^u = \langle x'^u_o, x'^u_z, y'^u, z'^u \rangle$ as shown in Figure 7, we will obtain a range of values for its MAC.

The convex range/interval of values of MAC for each such plant u when the technology is non-parametrically specified, will be denoted by $\mathcal{MAC}^u = \left[\ \underline{\gamma}^u, \ \bar{\gamma}^u \ \right]$, where the lower and upper bounds $\underline{\gamma}^u$ and $\bar{\gamma}^u$ will be defined more precisely below.

4.2.2 Non-parametric computation of MAC of a plant and its determinants: A primal approach.

The convex interval of values of MAC for each plant u that were obtained above using dual linear programming approach when the technology is non-parametrically represented can also be characterised using a primal linear programming approach using the concepts of one-sided directional derivatives and a sub differential.[15] To see this, let's first construct the frontiers of T_1 and T_2 of vintages $\hat{t}$ and t, respectively, in the non-parametric case as follows:

$$\begin{array}{cc}
F\left(\hat{t}, x_o, x_z\right) := \max_{y,\ \lambda}\ y & G\left(t, x_z\right) := \min_{z,\ \mu}\ z \\
\text{subject to} & \text{subject to} \\
\lambda^\top X_o^{\hat{t}} \leq x_o,\ \ \lambda^\top Y^{\hat{t}} \geq y,\ \ \lambda^\top X_z^{\hat{t}} \leq x_z & \mu^\top X_z^t \geq x_z,\ \ \mu^\top Z^t \leq z \\
\sum_{u=1}^U \lambda_u \leq 1,\ \ \lambda \geq 0_U & \sum_{u=1}^U \mu_u \leq 1,\ \ \mu \geq 0_U
\end{array}$$

The frontier of T_1 (respectively, T_2) is obtained by maximising the value of the good (respectively, bad) output subject to the production taking place on sub-technology T_1 (resepectively T_2). Under our maintained assumptions on T_1 and T_2, we obtain the following functional representation of these sets:

$$\begin{aligned}
T_1(\hat{t}) &= \{\langle x_o, x_z, y, z\rangle \in \mathbf{R}_+^{N_1+N_2+2} \mid y \leq F\left(\hat{t}, x_o, x_z\right)\} \\
T_2(t) &= \{\langle x_o, x_z, y, z\rangle \in \mathbf{R}_+^{N_1+N_2+2} \mid z \geq G\left(t, x_z\right)\}
\end{aligned} \tag{23}$$

Under our assumptions on T_1 and T_2, function F is concave and non-decreasing in all the inputs, while function G is convex and increasing in the emission-causing input. The DEA-non-parametric nature of the sub-technologies implies that functions F and G will not be smooth – their graphs will be piece-wise linear.

Define the right and the left partial derivatives of functions F and G with respect to x_z at

[15] See, for instance, Rockafellar 1997.

$\langle x_o, x_z \rangle \in \mathbf{R}^3_+$, respectively, as follows. These are also the limiting marginal products of the emission-causing input in the production of the good and bad outputs. They provide the lower and upper bounds of the marginal products indexed by LB and UB, respectively.

$$
\begin{aligned}
MP_1^{LB}\left(\hat{t}, x_o, x_z\right) &= \frac{\partial F\left(\hat{t}, x_o, x_z\right)}{\partial x_z}_{+} := \lim_{\Delta x_z \longrightarrow 0+} \frac{F\left(\hat{t}, x_o, x_z + \Delta x_z\right) - F\left(\hat{t}, x_o, x_z\right)}{\Delta x_z} \\
MP_1^{UB}\left(\hat{t}, x_o, x_z\right) &= \frac{\partial F\left(\hat{t}, x_o, x_z\right)}{\partial x_z}_{-} := \lim_{\Delta x_z \longrightarrow 0-} \frac{F\left(\hat{t}, x_o, x_z + \Delta x_z\right) - F\left(\hat{t}, x_o, x_z\right)}{\Delta x_z} \\
MP_2^{UB}\left(t, x_z\right) &= \frac{dG\left(t, x_z\right)}{dx_z}_{+} := \lim_{\Delta x_z \longrightarrow 0+} \frac{G\left(t, x_z + \Delta x_z\right) - G\left(t, x_z\right)}{\Delta x_z} \\
MP_2^{LB}\left(t, x_z\right) &= \frac{dG\left(t, x_z\right)}{dx_z}_{-} := \lim_{\Delta x_z \longrightarrow 0-} \frac{G\left(t, x_z + \Delta x_z\right) - G\left(t, x_z\right)}{\Delta x_z}
\end{aligned}
$$

From the concavity of function F and convexity of function G it follows that $MP_1^{LB}\left(\hat{t}, x_o, x_z\right) \leq MP_1^{UB}\left(\hat{t}, x_o, x_z\right)$ and $MP_2^{LB}\left(t, x_z\right) \leq MP_2^{UB}\left(t, x_z\right)$. Figure 7 illustrates this. The frontier of sub-technology T_1 (respectively, sub-technology T_1) is the graph of function F (respectively, graph of function G) in the space of x_z and y (respectively, x_z and z).

Suppose the vector of fixed inputs and the amount of the emission-causing input coal employed by a power plant u are x_o^u and x_z^u, respectively. Then upper and lower bounds of its MAC at the technically efficient point $\langle x_o^u,\ x_z^u,\ F\left(\hat{t}, x_o^u, x_z^u\right),\ G\left(t, x_z^u\right)\rangle$ of the technology $T(\hat{t}, t) = T_1(\hat{t}) \cap T_2(t)$ are

$$
\underline{\gamma}^u = \underline{\gamma}\left(\hat{t}, t, x_o^u, x_z^u\right) = \frac{MP_1^{LB}\left(\hat{t}, x_o^u, x_z^u\right)}{MP_2^{UB}\left(t, x_z^u\right)} \text{ and } \bar{\gamma}^u = \bar{\gamma}\left(\hat{t}, t, x_o^u, x_z^u\right) = \frac{MP_1^{UB}\left(\hat{t}, x_o^u, x_z^u\right)}{MP_2^{LB}\left(t, x_z^u\right)} \qquad (24)
$$

Hence, the convex range of MAC for plant u is obtained as the interval

$$
\mathcal{MAC}^u = \mathcal{MAC}\left(\hat{t}, t, x_o^u, x_z^u\right) = \left[\ \underline{\gamma}\left(\hat{t}, t, x_o^u, x_z^u\right),\ \bar{\gamma}\left(\hat{t}, t, x_o^u, x_z^u\right)\ \right] = \left[\ \underline{\gamma}^u,\ \bar{\gamma}^u\ \right]. \qquad (25)
$$

4.3 Allocative efficiency and the range of sectoral MACs: A non-parametric approach.

The non-parametric DEA analogue of problem (8) that results in allocative efficiency in the sector with emission cap z, the distribution of fixed inputs $\langle x_o^1, \ldots, x_o^U \rangle$, and sub-technologies

T_1 and T_2 of vintages $\hat{t}$ and t, respectively, is given as follows:[16]

$$\mathcal{Y}\left(\hat{t}, t, x_o^1, \ldots, x_o^U, z\right) = \max_{\{x_z^u, y^u, z^u, \lambda^u, \mu^u\}_{u=1}^U} \sum_{u=1}^U y^u$$
$$\text{subject to the following for all } u = 1, \ \ldots, \ U$$
$$\lambda^{u\top} X_o^{\hat{t}} \le x_o^u, \ \lambda^{u\top} Y^{\hat{t}} \ge y^u, \ \lambda^{u\top} X_z^{\hat{t}} \le x_z^u$$
$$\mu^{u\top} X_z^t \ge x_z^u, \ \mu^{u\top} Z^t \le z^u$$
$$\sum_{i=1}^U \lambda_i^u \le 1, \ \sum_{i=1}^U \mu_i^u \le 1, \ \lambda^u \ge 0_U, \mu^u \ge 0_U$$
$$\sum_{u=1}^U z^u \le z \tag{26}$$

Here, as in Section 3.5 for the parametric case, we maximise the sectoral output given the technology, the emission cap, and the distribution of fixed inputs. Suppose this problem has a unique solution. Denote the functions that give the optimal levels of emission, good output, and the emission-causing input for plant u, respectively, as

$$y^u = \mathscr{Y}^u\left(\hat{t}, t, x_o^1, \ldots, x_o^U, z\right), \ z^u = \mathfrak{Z}^u\left(\hat{t}, t, x_o^1, \ldots, x_o^U, z\right), \ x_z^u = \mathfrak{X}_z^u\left(\hat{t}, t, x_o^1, \ldots, x_o^U, z\right) \tag{27}$$

Definition 15 *An allocation $a = \{\langle x_o^u, \ x_z^u, \ y^u, \ z^u\rangle\}_{u=1}^U$ is allocatively efficient for the distribution of fixed inputs and emission cap $\langle x_o^1, \ldots, x_o^U, z\rangle$ and sub-technologies T_1 and T_2 of vintages $\hat{t}$ and t, respectively, if (27) is true.*[17]

As in the parametric case, when the sector operates with allocative efficiency, there will be an MAC value that will be common to all plants. Hence, define the intersection of ranges of plant-level MACs obtained when the sector operates with allocative efficient with the distribution of fixed inputs and emission cap $\langle x_o^1, \ldots, x_o^U, z\rangle$ and sub-technologies T_1 and T_2 of vintages $\hat{t}$ and t, respectively as

$$\mathfrak{MAC}\left(\hat{t}, t, x_o^1, \ldots, x_o^U, z\right) := \bigcap_{u=1}^U \mathcal{MAC}\left(\hat{t}, t, x_o^u, \mathfrak{X}_z^u\right) \ = \ \bigcap_{u=1}^U \left[\ \underline{\gamma}\left(\hat{t}, t, x_o^u, \mathfrak{X}_z^u\right), \ \bar{\gamma}\left(\hat{t}, t, x_o^u, \mathfrak{X}_z^u\right)\ \right] \tag{28}$$

where $\mathfrak{X}_z^u = \mathfrak{X}_z^u\left(\hat{t}, t, x_o^1, \ldots, x_o^U, z\right)$ for all $u = 1, \ldots, U$.

Remark 16 *When the sector operates with allocative efficiency, the intersection of the plant-level ranges of MACs is non-empty and defines a set of common/sectoral MACs in the non-parametric case:*

$$\mathfrak{MAC}\left(\hat{t}, t, x_o^1, \ldots, x_o^U, z\right) \neq \emptyset$$

[16] λ^u and μ^u are each $U \times 1$-dimensional intensity vectors corresponding to power plant u.

[17] If $a = \{\langle x_o^u, \ x_z^u, \ y^u, \ z^u\rangle\}_{u=1}^U$ is an allocatively efficient allocation corresponding to the distribution of fixed inputs and emission cap $\langle x_o^1, \ldots, x_o^U, z\rangle$ and sub-technologies T_1 and T_2 of vintages $\hat{t}$ and t, respectively, then $y^u = F\left(\hat{t}, x_o^u, x_z^u\right)$ and $z^u = G\left(t, x_z^u\right)$ for all $u = 1, \ldots, U$, *i.e.*, each plant also operates with technical efficiency.

For every distribution of fixed inputs and emission cap $\langle x_o^1, \ldots, x_o^U, z\rangle \in \mathbf{R}_+^{NU+1}$ and given sub-technologies T_1 and T_2 of vintages $\hat{t}$ and t, two elements of the range of sectoral MACs obtained under allocative efficiency, $\mathfrak{MAC}\left(\hat{t}, t, x_o^1, \ldots, x_o^U, z\right)$, can be systematically highlighted and computed as follows. Define the minimum of the upper bounds and the maximum of the lower bounds of the intervals of MACs of all plants at the allocatively efficient allocation as:

$$\begin{aligned}
\bar{\gamma}^{min}\left(\hat{t}, t, x_o^1, \ldots, x_o^U, z\right) &:= \min\{\bar{\gamma}\left(\hat{t}, t, x_o^1, \mathfrak{X}_z^1\right), \ \ldots, \ \bar{\gamma}\left(\hat{t}, t, x_o^U, \mathfrak{X}_z^U\right)\} \\
\underline{\gamma}^{max}\left(\hat{t}, t, x_o^1, \ldots, x_o^U, z\right) &:= \max\{\underline{\gamma}\left(\hat{t}, t, x_o^1, \mathfrak{X}_z^1\right), \ \ldots, \ \underline{\gamma}\left(\hat{t}, t, x_o^U, \mathfrak{X}_z^U\right)\},
\end{aligned}$$

where $\mathfrak{X}_z^u = \mathfrak{X}_z^u\left(\hat{t}, t, x_o^1, \ldots, x_o^U, z\right)$ for all $u = 1, \ldots, U$. The theorem below, proof of which can be found in the Appendix, states that the interval defined by $\underline{\gamma}^{max}$ and $\bar{\gamma}^{min}$ forms the set of all the sectoral/common MACs.

Theorem 17 *For every distribution of fixed inputs $\mathbf{x}_o = \langle x_o^1, \ldots, x_o^U\rangle \in \mathbf{R}_+^{NU}$, emission cap $z \geq 0$, and sub-technologies T_1 and T_2 of vintages $\hat{t}$ and t, respectively, we have $\underline{\gamma}^{max}\left(\hat{t}, t, \mathbf{x}_o, z\right) \leq \bar{\gamma}^{min}\left(\hat{t}, t, \mathbf{x}_o, z\right)$ and $\mathfrak{MAC}\left(\hat{t}, t, \mathbf{x}_o, z\right) = \left[\underline{\gamma}^{max}\left(\hat{t}, t, \mathbf{x}_o, z\right), \ \bar{\gamma}^{min}\left(\hat{t}, t, \mathbf{x}_o, z\right)\right]$.*

Given technology $T(\hat{t}, t)$ and a technically efficient (but possibly, allocatively inefficient) allocation $a_{TE} = \{\langle x_o^u, x_z^u, y^u, z^u\rangle\}_{u=1}^U$, define the following terms:[18]

$$\begin{aligned}
m^{min}\left(\hat{t}, t, \mathbf{x}_o, \mathbf{x}_z\right) &:= \min\{\underline{\gamma}\left(\hat{t}, t, x_o^1, x_z^1\right), \ \ldots, \ \underline{\gamma}\left(\hat{t}, t, x_o^U, x_z^U\right)\} \\
m^{max}\left(\hat{t}, t, \mathbf{x}_o, \mathbf{x}_z\right) &:= \max\{\underline{\gamma}\left(\hat{t}, t, x_o^1, x_z^1\right), \ \ldots, \ \underline{\gamma}\left(\hat{t}, t, x_o^U, x_z^U\right)\} \\
M^{max}\left(\hat{t}, t, \mathbf{x}_o, \mathbf{x}_z\right) &:= \max\{\bar{\gamma}\left(\hat{t}, t, x_o^1, x_z^1\right), \ \ldots, \ \bar{\gamma}\left(\hat{t}, t, x_o^U, x_z^U\right)\}
\end{aligned} \tag{29}$$

where $\mathbf{x}_o = \langle x_o^1, \ldots, x_o^U\rangle$ and $\mathbf{x}_z = \langle x_z^1, \ldots, x_z^U\rangle$. The functions m^{min} and m^{max} (respectively, functions M^{min} and M^{max}) provide the minimum and maximum of the lower (respectively, upper) bounds of ranges of MACs of plants at the technically efficient allocation a_{TE}. Analogous to Lemma 8 in the parametric case the range of sectoral MACs of the sector described above lies in the interval that contains the MACs of all plants at a_{TE}:

$$\mathfrak{MAC}\left(\hat{t}, t, \mathbf{x}_o, z\right) \subseteq \left[m^{min}\left(\hat{t}, t, \mathbf{x}_o, \mathbf{x}_z\right), M^{max}\left(\hat{t}, t, \mathbf{x}_o, \mathbf{x}_z\right)\right].$$

4.4 Decomposition of a change in the sectoral MAC.

The range of sectoral MACs can change over time due to changes in the distribution of fixed inputs or the emission cap or due to technological change. To decompose and study this change using the non-parametric approach, we adopt the following notation that involves triple subscripts – the first subscript indicates the vintage of the technology being considered, the second subscript indicates either the time point of the distribution of fixed inputs or the distribution of fixed inputs itself that is under consideration, while the third subscript indicates either the time point of the emission cap or the emission cap itself that is under consideration. Thus,

[18] Technical efficiency of allocation a implies that $y^u = f\left(\hat{t}, x_o^u, x_z^u\right)$ and $z^u = g\left(t, x_z^u\right)$ for all $u = 1, \ldots, U$.

$\bar{\gamma}^{min}_{T(t,t),\hat{t},t} := \bar{\gamma}^{min}\left(t, t, x^1_o(\hat{t}), \ldots, x^U_o(\hat{t}), z(t)\right)$, is the maximum value of the sectoral MAC achieved (see Theorem 17) when the technology is such that both its sub-technologies are of vintage t, the distribution of fixed inputs is that of year $\hat{t}$, and the emission cap is that of year t. While the underlying technologies in $\bar{\gamma}^{min}_{T(t,t),\bar{\mathbf{x}}_o,\bar{z}}$ and $\bar{\gamma}^{min}_{T(t,t),\hat{t},t}$ are the same, the distribution of fixed inputs and the emission cap in the former are $\bar{\mathbf{x}}_o = \langle \bar{x}^1_o, \ldots, \bar{x}^U_o \rangle$ and $\bar{z}$, respectively. Similarly, we can define $\mathfrak{X}^u_{z_{T(t,t),\hat{t},t}} := \mathfrak{X}^u_z\left(t, t, x^1_o(\hat{t}), \ldots, x^U_o(\hat{t}), z(t)\right)$ and $\mathfrak{X}^u_{z_{T(t,t),\bar{\mathbf{x}}_o,\bar{z}}} := \mathfrak{X}^u_z\left(t, t, \bar{x}^1_o, \ldots, \bar{x}^U_o, \bar{z}\right)$, *etc.*

The total change in the lower or upper bounds of the range of sectoral MACs between time point $\hat{t}$ (the base year) and t (the current year) can be decomposed as follows:

$$\begin{aligned}
\bar{\gamma}^{min}_{T(t,t),t,t} - \bar{\gamma}^{min}_{T(\hat{t},\hat{t}),\hat{t},\hat{t}} &= \left[\bar{\gamma}^{min}_{T(\hat{t},\hat{t}),t,\hat{t}} - \bar{\gamma}^{min}_{T(\hat{t},\hat{t}),\hat{t},\hat{t}}\right] + \left[\bar{\gamma}^{min}_{T(\hat{t},\hat{t}),t,t} - \bar{\gamma}^{min}_{T(\hat{t},\hat{t}),t,\hat{t}}\right] + \left[\bar{\gamma}^{min}_{T(t,t),t,t} - \bar{\gamma}^{min}_{T(\hat{t},\hat{t}),t,t}\right] \\
\underline{\gamma}^{max}_{T(t,t),t,t} - \underline{\gamma}^{max}_{T(\hat{t},\hat{t}),\hat{t},\hat{t}} &= \left[\underline{\gamma}^{max}_{T(\hat{t},\hat{t}),t,\hat{t}} - \underline{\gamma}^{max}_{T(\hat{t},\hat{t}),\hat{t},\hat{t}}\right] + \left[\underline{\gamma}^{max}_{T(\hat{t},\hat{t}),t,t} - \underline{\gamma}^{max}_{T(\hat{t},\hat{t}),t,\hat{t}}\right] + \left[\underline{\gamma}^{max}_{T(t,t),t,t} - \underline{\gamma}^{max}_{T(\hat{t},\hat{t}),t,t}\right]
\end{aligned} \tag{30}$$

Starting from year $\hat{t}$, the first terms on the right of either equations in (30) refer to the changes in the upper and lower bounds of the sectoral MAC due to change in the distribution of the fixed inputs to its year t level, while holding the emission cap and the technology fixed at their year $\hat{t}$ levels. Starting from here, the second terms compute the change in the upper and lower bounds of the sectoral MAC when the emission cap also changes to its year t level with no change in the technology; while the third terms refer to the remaining changes in the lower and upper bounds of the sectoral MAC when the technology also changes to the one prevailing in year t. Our empirical analysis will systematically study this decomposition in the case of the Indian thermal power sector in a step-wise manner.

5 Data and Results.

5.1 Data.

The study uses annual data on 48 coal-fired thermal power plants for the years 2014-15 and 2009-2010, of which 46 are included in the data for each of the two years.[19] These plants are run by 16 major power generating companies belonging to Centre, state and private sector.

The data is on the intended output, net electricity, measured in gigawatt hours (GWh); the unintended output, CO_2 emission, measured in metric tonnes (MT); the emission-causing input, aggregate heat from coal and oil consumption by the coal-based thermal power plants, measured in millions of kilocalories (mill of Kcal); and two non-emission causing inputs, plant capacity, measured in megawatt (MW) and plant operating availability (POA), measured in megawatt hours (MWh). Following works such as Behera et al (2010) and Sahoo et al (2017), plant capacity here is taken as a proxy for capital as data on capital employed by power plants is not systematically available. Likewise, following several recent studies such as Sueyoshi and Goto (2010, 2012) and Sahoo et al. (2017) that have suggested use of managerial inputs, we

[19]The cross-sections of plants considered in each of the two years are not identical.

have included POA as a managerial input in our study. It is defined as the percentage of total capacity that is available to the plant for electricity generation after deducting the percentage lost due to forced outage and planned maintenance. The contribution of labour in the Indian thermal power sector is not significant and hence is not modelled (see Kumar et al 2015), especially in the absence of coherent data on labour employed in this sector.[20]

Data on these variables are collected from several publications of Central Electricity Authority (CEA). Table 1 provides the descriptive statistic of the variables used in this study.

Table 1: Descriptive statistics of the data

	2010				2015			
	Min	Max	Mean	Stdev	Min	Max	Mean	Stdev
Net electricity generation in GWh	554.2	25898	7723.8	5536.557	485.1	27594	8576.7	6158.608
CO2 emission in MT	903986.1504	24812326	8103915.145	5168077.879	966999.4949	26548438	8782090.232	5750888.077
Capacity in MW	135	3260	1176.1	695.268	240	4260	1471.3	827.946
Aggregate heat in mill of Kcal	2384364.0	65433636.2	21487640.4	13579679.001	2397060.0	70859304.5	23099027.0	15317496.996
POA in % of MWh	43.0	97.0	86.5	10.030	34.6	97.5	78.2	17.590

5.2 Results.

The non-parametric methodology developed in Section 4 is applied in our empirical work. Measurement of MAC requires that plants operate on the frontier of the technology. But the allocations directly supported by the data are not technically efficient viz-a-viz the frontiers of the non-parametric DEA by-production technologies prevailing in years $\hat{t} = 2010$ (which can be considered as the base year) and $t = 2015$ (which can be considered as the current year). In each of these years, holding input usage by all plants fixed at the levels of the data, the technology permits increases in electricity generation and reductions in CO_2 emission of most plants beyond the levels specified by the data. Doing so for each year leads us to the technically efficient allocations corresponding to the two years with input usage fixed as per the data for these years. We denote these allocations by $a_{TE_{\hat{t}}}$ and a_{TE_t}, respectively. Table 2 shows that, at these allocations, the total sectoral emissions are 351995315.1 MT and 400649339.5 MT for the years 2010 and 2015, respectively. We consider these as the sectoral emission caps prevailing in these two years. The table also provides the total sectoral electricity generation at allocations $a_{TE_{\hat{t}}}$ and a_{TE_t}.

[20]For greater details on the dataset, the reader is referred to Murty and Nagpal (2019).

Table 2: Sectoral electricity and emission generation and heat input usage under data and technical efficiency (TE)

	2010		2015	
	Data	TE	Data	TE
Net electricity generation in GWh	355292.60	416253.01	394526.43	450210.65
CO2 emission in MT	372780096.68	351995315.10	403976150.66	400649339.50
Heat input in mill of Kcal	988431456.23	1046778652.09	1062555242.79	1106190479.40

Allocations $a_{TE_{\hat{t}}}$ and a_{TE_t} are by no means allocatively efficient. Table A1 in the appendix demonstrates that the intersection of the ranges of MACs of individual plants in these two years is empty – there is no value of MAC that is common to all plants in each of these two years. Let us denote the allocations that are obtained when the sector operates under allocative efficiency with the technologies, the distributions of fixed-input, and the emission caps of the base and current years as $a_{AE_{\hat{t}}}$ and a_{AE_t}, respectively. Table A2 in the Appendix shows that, in each of these two years, the sectoral MAC is unique – the intersection of ranges of MACs of individual plants in the sector is a singleton in each of the two years.[21] These are reported in Table 3, and one can infer that the sectoral MAC has increased during this period from $\underline{\gamma}^{max}_{T(\hat{t},\hat{t}),\hat{t},\hat{t}} = \bar{\gamma}^{min}_{T(\hat{t},\hat{t}),\hat{t},\hat{t}} = 914.2$ KWh/MT in $\hat{t} = 2010$ to $\underline{\gamma}^{max}_{T(t,t),t,t} = \bar{\gamma}^{min}_{T(t,t),t,t} = 1046.6$ KWh/MT in $t = 2015$.

Table 3: Decomposition of change in sectoral MAC between 2010 and 2015 (KWh/MT).

AE/TE	T_1	T_2	POA	Capacity	Emission cap year	Emission cap value	x_z fixed	Sectoral MAC LB	Sectoral MAC UB	m^{max}	M^{max}	m^{min}
AE_{2010}	2010	2010	2010	2010	2010	351995315.1	No	914.2	914.2			
$AE_{1.1}$	2010	2010	2015	2010	2010	351995315.1	No	861.4	861.4			
AE_1	2010	2010	2015	2015	2010	351995315.1	No	977.7	977.7			
AE_2	2010	2010	2015	2015	2015	400649339.5	No	977.7	977.7			
TE_1	2010	2015	2015	2015		425875020.0	AE_2 level			1043.7	1187.0	489.6
AE_3	2010	2015	2015	2015		425875020.0	No	1043.7	1043.7			
AE_4	2010	2015	2015	2015	2015	400649339.5	No	1043.7	1043.7			
TE_2	2015	2015	2015	2015	2015	400649339.5	AE_4 level			1046.6	1046.6	0.0
AE_{2015}	2015	2015	2015	2015	2015	400649339.5	No	1046.6	1046.6			

Table 3 traces the decomposition of the change in the sectoral MAC due to the changes in the distributions of fixed inputs (the plant capacity and POA), the emission cap, and the two sub-technologies T_1 and T_2 as we move from allocation $a_{AE_{\hat{t}}}$ to allocation a_{AE_t}.

5.2.1 Change in the sectoral MAC due to changes in the distributions of fixed inputs and the emission cap.

In our case, the fixed inputs are plant capacity and POA. Table 1 shows that while the average POA has fallen from 86.5% to 78.2% between 2010 and 2015, average plant capacity has increased from 1176 MW to 1471 MW in this period. The table also shows an increase in the total capacity of the sector during this period. Table 4 notes that the distribution of the managerial input POA has deteriorated with the share of plants with low managerial input increasing and the share of plants with high managerial input decreasing over this period. Share of plants with managerial input greater than 70% fell from 93% in 2010 to 78% in 2015. A reverse trend is

[21]That is, $\left[\underline{\gamma}^{max}_{T(\hat{t},\hat{t}),\hat{t},\hat{t}}, \bar{\gamma}^{min}_{T(\hat{t},\hat{t}),\hat{t},\hat{t}}\right]$ and $\left[\underline{\gamma}^{max}_{T(t,t),t,t}, \bar{\gamma}^{min}_{T(t,t),t,t}\right]$ contain unique values.

seen in the case of plant capacity. The share of plants with low capacity fell and the share of plants with capacity greater than 1000 MW increased from 50% in 2010 to 72% in 2015.

Table 4: Distributions of POA and capacity

POA	2010	2015	Capacity	2010	2015
0 - 40	0	4	0 - 500	8	6
41 - 70	3	6	501 - 1000	15	7
71 - 90	25	24	1001 - 1500	13	17
91 - 100	18	12	1501 - 2000	5	7
			2501 - 3500	5	8
			3501-	0	1
Share of plants with POA above 70%	94%	78%	Share of plants with capacity above 1000 MW	50%	72%
Change in MP due to change in distribution of POA from 2010 to 2015 level			Change in MP due to change in distribution of capacity and POA from 2010 to 2015 level		
	2010	2015		2010	2015
Mean MP_1^{LB}	296.6	265.5	Mean MP_1^{LB}	265.5	339.1
Mean MP_1^{UB}	396.6	364.2	Mean MP_1^{UB}	364.2	392.8
Mean MP_2^{LB}	0.367	0.367	Mean MP_2^{LB}	0.367	0.378
Mean MP_2^{UB}	0.367	0.367	Mean MP_2^{UB}	0.367	0.386

The impact of these changes in the distribution of fixed inputs on the sectoral MAC is noted in Tables 3 and 4. In Table 3, $a_{AE_{1.1}}$ denotes the allocation that is obtained when the sector operates with allocative efficiency with the technology, emission cap, and distribution of capacity held fixed at the base year 2010 levels, but when the distribution of POA alone is allowed to change to the one prevailing in current year 2015. Starting from $a_{AE_{1.1}}$, allocation a_{AE_1} is obtained when the distribution of capacity is also additionally allowed to change to its level in year 2015.

Analogous to Assumption 9 in the parametric case, our empirical results indicate that the two fixed inputs do share a complementarity with the variable input, heat. Table 4 shows that with the deterioration in the managerial input POA during the period 2010 to 2015, both the lower and the upper bounds of the marginal productivity of the heat input in the production of the good output, MP_1, fall. For example, the mean value of the upper bound of MP_1 falls from 396.6 KWh/mill of Kcal to 364.3 KWh/mill of Kcal. Recalling that the MAC of a plant is the ratio of the marginal productivities of the heat input in the production of the good and bad outputs, this fall in MP_1 is responsible for the fall in the sectoral MAC seen in Table 3 from 914.2 KWh/MT under allocation $a_{AE_{\hat{t}}}$ to 861.4 KWh/MT under allocation $a_{AE_{1.1}}$.

However, with the improvement in the distribution of capacity between 2010 and 2015, MP_1 rises. For example, Table 4 shows that the mean upper bound on MP_1 rises from 364.3 KWh/mill of Kcal under allocation $a_{AE_{1.1}}$ to 392.8 KWh/mill of Kcal under allocation a_{AE_1}. Due to this, as seen in Table 3, the sectoral MAC rises from 861.4 KWh/MT under allocation $a_{AE_{1.1}}$ to 977.7 KWh/MT under allocation a_{AE_1}. These results are in line with Proposition 10 in the parametric case.

The net effect of change in the distributions of our two fixed inputs during the period $\hat{t} = 2010$ to $t = 2015$ is an increase in the sectoral MAC: It increased from $\underline{\gamma}^{max}_{T(\hat{t},\hat{t}),\hat{t},\hat{t}} = \bar{\gamma}^{min}_{T(\hat{t},\hat{t}),\hat{t},\hat{t}} =$ 914.2 KWh/MT under allocation $a_{AE_{\hat{t}}}$ to $\underline{\gamma}^{max}_{T(\hat{t},t),t,\hat{t}} = \bar{\gamma}^{min}_{T(\hat{t},t),t,\hat{t}} = 977.7$ KWh/MT under allocation a_{AE_1}.

It was noted above that during the period $\hat{t} = 2010$ to $t = 2015$ the emission cap increased from $z(\hat{t}) = 351995315.1$ MT to $z(t) = 400649339.5$ MT. Thus, as per the predictions of our theory (see Proposition 11), adapted to the non-parametric case, ceteris-paribus, an increase in the emission cap should not result in an increase in the sectoral MAC. It should either fall or remain the same. Table 3 shows that this change in the emission cap does not result in a further change in the sectoral MAC. Starting from allocatively efficient allocation a_{AE_1}, where the distributions of fixed inputs were allowed to change to their 2015 levels, the sectoral MAC continues to be 977.7 KWh/MT $= \underline{\gamma}^{max}_{T(\hat{t},\hat{t}),t,t} = \bar{\gamma}^{min}_{T(\hat{t},\hat{t}),t,t}$ at the allocatively efficient allocation a_{AE_2}, where the emission cap is also allowed to change to the 2015 level.

5.2.2 Change in the sectoral MAC due to change in sub-technology T_2.

Analogous to Step (i) in *Stage 1* in Section 3.6.4, starting from allocation a_{AE_2}, we move to the technically efficient allocation a_{TE_1} where each plant operates with the levels of its inputs prevailing at allocation a_{AE_2} and note the change in the frontier of sub-technology T_2 (both the shift in its level and the change in its slope, MP_2) when the vintage of sub-technology T_2 changes from $\hat{t} = 2010$ to $t = 2015$ holding sub-technology T_1 fixed at its $\hat{t} = 2010$ level. These and their consequences are studied below.

In Table 5, z_{TE}/z_{AE} and $MP_{2_{TE}}/MP_{2_{AE}}$ denote, respectively, the ratio of the minimum level of emission and the ratio of the marginal product of the heat input in the generation of CO_2 emission for a plant under allocations a_{TE_1} and a_{AE_2}.

Table 5: Impact of technical change in T_2

	MP_{2TE}/MP_{2AE}		z_{TE}/z_{AE}	MAC_{LB}	
	LB	UB		TE	AE
Minimum	0.937	0.937	0.998	489.6	585.8
Maximum	1.231	1.231	1.231	1043.7	977.7
Mean	0.993	0.982	1.096	878.4	851.2
Median	0.937	0.937	1.093	919.6	861.4
Std dev	0.099	0.089	0.061	148.3	98.9

Share of plants with $MP_{2TE}/MP_{2AE} \leq 1$ (LB)	Share of plants with $z_{TE}/z_{AE} \geq 1$	Share of plants for which LB of MAC increased in the move from AE to TE
76%	93%	76%

Table 5 shows that the mean value of the ratio z_{TE}/z_{AE} is 1.096 and that it is greater than one for 93% of the plants, indicating a dirty technological change around the level of the heat input usage at allocation a_{AE_2} for a majority of the plants. Hence, it is not surprising that the sectoral emission level rises when we move from allocation a_{AE_2} to allocation a_{TE_1}, precisely, Table 3 shows that the emission cap increases from $z(t) = 400649339.5$ MT to $\bar{z} = 425875020.0$ MT in this move.

A value lesser than one for $MP^u_{2_{TE}}/MP^u_{2_{AE}}$ implies that the technological change has led to a decline in the slope of the frontier of T_2 in the local region around the levels of the heat input usage in allocation a_{AE_2}. Table 5 shows that the mean values taken by this ratio for the lower and upper bounds are 0.993 MT/mill of Kcal and 0.982 MT/mill of Kcal, respectively, and for more than 70% of the plants this ratio is less than or equal to one.

Hence, we conclude that that a majority of plants have experienced a locally decreasingly dirty technological change between the period 2010 and 2015 – frontier of T_2 has shifted up, while its slope has fallen in the region where these plants are operating. While there are also plants for which the technological change has been increasingly dirty.

Since sub-technology T_1 does not change from its 2010 level in the move from allocation a_{AE_2} to a_{TE_1}, the plant-level MP_1s do not change in this move. Hence, since the MAC of a plant is a ratio of its MP_1 and MP_2, the consequence of the change in sub-technology T_2 is a rise in the MACs of plants whose MP_2 fell in the move from allocation a_{AE_2} to a_{TE_1}. In Table 5, the share of plants whose lower bounds on MP_2s fell is also the share of plants whose the lower bounds on MACs rose.

By construction, allocation a_{TE_1} is technically efficient. But it is not allocatively efficient. There is no non-empty intersection of the ranges of MACs of all plants at this allocation – there is no common value of MAC for all plants at this allocation. Table 3 shows that the unique sectoral MAC at the allocatively efficient allocation a_{AE_2} given by $\underline{\gamma}^{max}_{T(\hat{t},\hat{t}),t,t} = \bar{\gamma}^{min}_{T(\hat{t},\hat{t}),t,t} = 977.65$ KWh/MT lies to the left of the interval $[m^{max}, M^{max}] = [1043.7, 1187.0]$, where m^{max} and M^{max} are the maximum of the lower and upper bounds of the range of MACs prevailing at a_{TE_1}.

The technically efficient allocation a_{TE_1} generates $\bar{z}$ amount of sectoral emission using the new vintage t of sub-technology T_2. Let a_{AE_3} denote the allocation that can be achieved under allocative efficiency with vintage t of sub-technology T_2 and sectoral emission cap $\bar{z}$. Table 3 shows that the sectoral MAC under allocation a_{AE_3} is unique and given by $\underline{\gamma}^{max}_{T(\hat{t},t),t,\bar{z}} = \bar{\gamma}^{min}_{T(\hat{t},t),t,\bar{z}} = 1043.7$ KWh/MT. A non-parametric analogue of Lemma 8 in the parametric case would imply that this sectoral MAC at a_{AE_3} should lie in the range of MACs of individual plants at the technically efficient allocation a_{TE_1}. This is true, as we find that it lies in the interval $[m^{max}, M^{max}] = [1043.7, 1187.0]$, which is tighter than this range. Since the unique common sectoral MAC obtained at allocation a_{AE_2} prior to the change in sub-technology T_2 lies to the left of this interval, the sectoral MAC at allocation a_{AE_2} is less than the sectoral MAC at allocation a_{AE_3}.

$$977.65 \text{ KWh/MT} \; = \; \underline{\gamma}^{max}_{T(\hat{t},\hat{t}),t,t} = \bar{\gamma}^{min}_{T(\hat{t},\hat{t}),t,t} \; < \; \underline{\gamma}^{max}_{T(\hat{t},t),t,\bar{z}} = \bar{\gamma}^{min}_{T(\hat{t},t),t,\bar{z}} \; = \; 1043.7 \text{ KWh/MT}$$

But the emission-cap at allocation a_{AE_3}, $\bar{z}$, is greater than the emission cap $z(t)$ in the current period $t = 2015$, though at this allocation the sub-technology T_2 and the distribution of fixed inputs are of period t. So now we consider the move from allocation a_{AE_3} to allocation a_{AE_4}, where we revert back to emission cap $z(t)$. In the non-parametric case, a weak inequality version of the law of cap-induced diminishing sectoral MAC (see Proposition 11) can be expected to apply. In corroboration with this, since the emission cap is lower in allocation a_{AE_4} than in allocation a_{AE_3}, we find that the unique sectoral MAC does not fall. In fact, it remains the same as in allocation a_{AE_3}, namely 1043.7 KWh/MT.

Thus, in the move from the allocatively efficient allocation $a_{AE_{\hat{t}}}$ in the base period $\hat{t} = 2010$

to the allocatively efficient allocation a_{AE_4}, the emission cap, the distribution of fixed inputs, and the vintage of sub-technology T_2 were changed to those in the current period $t = 2015$ and the consequent impact on the sectoral MAC was noted. However, sub-technology T_1 continues to be of vintage $\hat{t}$.

5.2.3 Change in the sectoral MAC due to change in sub-technology T_1.

To study the impact of the change in sub-technology T_1 to its current year level on the sectoral MAC, we proceed as in *Stage 2* in Section 3.6.4. Starting from the allocatively efficient allocation a_{AE_4}, we move to the technically efficient allocation a_{TE_2}, where each plant operates with levels of inputs that it was using in the efficient allocation a_{AE_4} and note the change in the frontier of sub-technology T_1 (both the shift in its level and the change in its slope, MP_1) when the vintage of sub-technology T_1 changes to $t = 2015$ from $\hat{t} = 2010$ holding the vintage of sub-technology T_2 fixed at $t = 2015$.

Table 6: Impact of technical change in T_1

	MP_{1TE}/MP_{1AE}		y_{TE}/y_{AE}	MAC_{LB}	
	LB	UB		TE	AE
Minimum	0.000	0.000	0.959	0.0	658.4
Maximum	1.003	1.003	1.034	1046.6	1043.7
Mean	0.305	0.297	0.996	311.5	932.0
Median	0.000	0.000	0.987	0.0	919.6
Std dev	0.430	0.422	0.024	443.1	103.3

Share of plants with $MP_{1TE}/MP_{1AE} \leq 1$ (LB)	Share of plants for which LB of MAC decreased in the move from AE to TE
83%	83%

Table 6 shows that there is no major shift in the level of the frontier of T_1 due to technological change in sub-technology T_1.The descriptive statistics of the ratio of electricity generation post and prior to change in sub-technology T_1, denoted by y_{TE}/y_{AE} reveal that this ratio is very close to one for all plants.

Table 6 also shows that for most plants the ratio of the marginal productivity of the heat input in the production of electricity at allocations a_{TE_2} and a_{AE_4}, denote by $MP_{1_{TE}}/MP_{1_{AE}}$, has declined for a majority of the plants. For these plants the technological change in sub-technology T_1 has led to a decrease in MP_1. The table indicates that for 17% of the plants the ratio $MP_{1_{TE}}/MP_{1_{AE}}$ has increased. Hence for these plants, the technological change in sub-technology T_1 has led to an increase in MP_1. Since there is no change in sub-technology T_2 and usage of the emission-causing inputs has not changed for all plants in the move from a_{AE_4} to a_{TE_2}, there is no change in their MP_2s. Hence, recalling that MAC of a plant is the ratio of its MP_1 and MP_2, we find that, at allocation a_{TE_2}, there are plants for which MAC is higher than the sectoral MAC at allocation a_{AE_4} and there are plants for which the MAC is lower than the sectoral MAC at allocation a_{AE_4}. Thus, the sectoral MAC at a_{AE_4} lies in the range of MACs obtained at the technically efficient allocation a_{TE_2}. Within this range, Table 3 also shows that the unique sectoral MAC at the allocatively efficient allocation a_{AE_4} (given by $\underline{\gamma}^{max}_{T(\hat{t},t),t,t}$, $\bar{\gamma}^{min}_{T(\hat{t},t),t,t} = 1043.7$ KWh/MT) lies to the left of the interval $[m^{max}, M^{max}]$.

Allocation a_{TE_2} is only technically efficient under the technology, emission-cap and distribution of fixed inputs prevailing in the current year $t = 2015$. So we not consider the move to the allocation a_{AE_t} that will also be allocatively efficient under these specifications. Table 3 shows that the unique sectoral MAC at allocation a_{AE_t} is

$$\underline{\gamma}^{max}_{T(t,t),t,t} = \bar{\gamma}^{min}_{T(t,t),t,t} = 1046.6 \text{ KWh/MT } = m^{max} = M^{max} > 1043.7 = \underline{\gamma}^{max}_{T(\hat{t},t),t,t} = \bar{\gamma}^{min}_{T(\hat{t},t),t,t}.$$

Thus, it lies in the interval $[m^{max}, M^{max}]$ and is hence bigger than the sectoral MAC prevailing in the allocatively efficient allocation a_{AE_4}, when the vintage of sub-technology T_1 was still $\hat{t} = 2010$.

5.2.4 Contributions to changes in the sectoral MAC.

Table 7 gives the contributions of changes in the distribution of fixed inputs, the emission cap, and the technology to the changes in the sectoral MAC from the base year 2010 to the current year 2015 for the Indian thermal power sector.

Table 7: Contributions to changes in sectoral MAC in KWh/MT

			Level	%
Net contribution of change in fixed inputs			63.5	47.96%
	Level	%		
Contribution of change in fixed input POA	-52.8	-39.88%		
Contribution of change in fixed input plant capacity	116.3	87.84%		
Contribution of change in emission cap			0	0%
Contribution of change in T_2			66	49.85%
Contribution of change in T_1			2.9	2.19%
Total contributions			132.4	100%
	level	%		
Total change in sectoral MAC between 2010 and 2015	132.4	14.48%		

The table shows that there is a 14.48% increase in the sectoral MAC between 2010 to 2015. It changed from 914.2 KWh/MT in 2010 to 1046.6 KWh/MT in 2015 by an amount 132.4 KWh/MT. Table 7 shows that change in sub-technology T_2 followed by change in the distribution of fixed inputs are the major contributors towards this change, with the former contributing to around 50% and the latter to around 48% of the change. The change in sub-technology T_1 contributed to around 2% of the total change in the sectoral MAC, while the change in the emission cap had no impact on it.

6 A discussion and conclusions.

In this work we have presented a general theory of how changes in the productive environment in which firms act, such as a technological change, changes in the distributions of fixed inputs, or a change in the emission cap, impact the plant-level and the sectoral MAC. The theory is based on the understanding of the MAC of a plant as the ratio of MP_1 and MP_2, which are, respectively, the marginal productivities of the emission-causing input in the production of what is valuable to the firm (in our case the production of the good output, though it could also be profit or some other objective) and the production of the emission. What this ratio signifies is that to cut a unit of emission requires a reduction in the usage of an emission-generating input such as a fossil fuel (given by the inverse of MP_2). But this adversely affects the production of the good output such as electricity. This reduction in the production of the good output due to an extra unit of abatement (given by MP_1/MP_2) is a measure of its MAC. From this it becomes clear that changes in the productive environment that increase MP_1, imply a greater loss of good-output production when emission is reduced by an extra unit – when the marginal productivity of a fossil fuel in the generation of electricity increases due to a technological change or an increase in the capacity of a power plant, it has a lot more to lose when it undertakes abatement.

Furthermore, any change in the productive environment of a plant such as a technological change that decreases (respectively, increases) MP_2 will tend to increase (respectively, decrease) MAC, as then it becomes harder (respectively, easier) for the plant to reduce emission – *e.g.,* when the marginal productivity of a fossil fuel in generating the emission decreases due to an increasingly clean technological change, then a greater reduction in the fossil fuel and a concomitant greater reduction in good-output production will be required to achieve a unit reduction in emission. But, a technological change in the production of the bad output impacts MAC also by impacting the level of the bad-output frontier. A clean (respectively, dirty) technological change implies a lower (respectively, a higher) level of emission generation at every level of usage of the emission-causing input. Hence, a given level of abatement can now be achieved with the new technology with a higher (respectively, a lower) usage of the emission-causing input, causing diminishing returns in electricity generation (due to concavity of the good output frontier) and increasing returns in emission generation (due to the convexity of the bad output frontier) to kick in. These tend to reduce MP_1 and increase MP_2 (respectively, increase MP_1 and reduce MP_2), causing the MAC to decrease (respectively, increase). Thus a technological change in bad-output production impacts the MAC by impacting both the level and the slope (MP_2) of the bad output frontier. We show that, in the case of increasingly clean or increasingly dirty technological changes, these two effects work in the opposite directions so that the net effect on a plant's MAC is ambiguous, while in the case of decreasingly clean or decreasingly dirty technological changes, they tend to reinforce each other in impacting the MAC of the plant (negatively in the former case and positively in the latter).

Hence, it is highly likely (as seen in some of our examples) that increasingly clean technological changes can lead to increases in total and marginal costs of abatement, while increasingly dirty technological changes can lower them. Thus, clean technological changes are neither necessary nor sufficient means for achieving lower abatement costs. Measuring gains from policy induced technological change is more complex when the MAC schedule does not globally shift down. Our work indicates the need for a production function approach for computing the gains from ITC.

Our non-parametric empirical analysis shows that real-life technological changes need not be clean and in line with the climate-change objectives of the policy maker. Production modelling approaches need to be flexible enough to capture any type of technological change and stand ready to reveal the true nature of such changes embedded in a given dataset, so that a true assessment of the real-life technological changes and their impacts can be made.

It is important, as implied by other works in the literature, to understand the impacts of technological change on MAC in a more extended production model than the one presented here that allows for alternative sources of energy and inter-fuel substitutability. That is the agenda of an on-going research in the framework developed in this paper. For example, our framework predicts and explains how an increase in the usage of a renewable source of energy can reduce MAC (cause the MAC schedule to shift down). Precisely, it does so by reducing the slope of the good output frontier, MP_1, which gives the loss in electricity generation due to a unit reduction in the usage of the fossil fuel. MP_1 will fall when the usage of a renewable input is increased, as the renewable acts as a substitute for the fossil fuel and reduces the loss in electricity generation that follows a unit reduction in the usage of the fossil fuel. We conjecture that the impact of a technological change that saves on a clean (renewable) fuel on the MAC could be ambiguous.

Appendix

Proof. Proposition 5 We will prove Part (i) of the proposition below. Part (ii) can be similarly proved.

First we show that, under the conditions of the proposition, we have $MP_2(\mathcal{B},0) > MP_2(\bar{\mathcal{B}},0)$ when $\bar{\mathcal{B}} > \mathcal{B}$. Suppose not. Then, under the maintained assumptions, $MP_2(\mathcal{B},0) < MP_2(\bar{\mathcal{B}},0)$. Let $\alpha = MP_2(\bar{\mathcal{B}},0) - MP_2(\mathcal{B},0)$. Then $\alpha > 0$. Pick an $\epsilon \in (0,\alpha)$. Then $N_\epsilon(\alpha) \subset \mathbf{R}_{++}$, where $N_\epsilon(\alpha)$ denotes the ϵ neighbourhood of α. The continuity of function MP_2 under our maintained assumptions implies that there exists $\eta > 0$ such that $MP_2(\bar{\mathcal{B}},x_z) - MP_2(\mathcal{B},x_z) \in N_\epsilon(\alpha)$ whenever $x_z \in N_\eta(0) \cap \mathbf{R}_{++}$. Hence, $MP_2(\bar{\mathcal{B}},x_z) - MP_2(\mathcal{B},x_z) > 0$ whenever $x_z \in N_\eta(0) \cap \mathbf{R}_{++}$. Hence,

$$\int_0^\eta \left[MP_2(\bar{\mathcal{B}},x_z) - MP_2(\mathcal{B},x_z)\right] dx_z = \int_0^\eta \frac{\partial g(\bar{\mathcal{B}},x_z)}{\partial x_z} dx_z - \int_0^\eta \frac{\partial g(\mathcal{B},x_z)}{\partial x_z} dx_z > 0$$
$$\implies g(\bar{\mathcal{B}},\eta) > g(\mathcal{B},\eta)$$

which is a contradiction to the assumption that the technological change in T_2 is globally clean.

Now redefine $\alpha = MP_2(\mathcal{B},0) - MP_2(\bar{\mathcal{B}},0)$. Then $\alpha > 0$. Pick $\epsilon \in (0,\alpha)$. Again using the continuity of $MP_2(\mathcal{B},x_z) - MP_2(\bar{\mathcal{B}},x_z)$, we can show that there exists $\delta > 0$ such that $MP_2(\mathcal{B},x_z) - MP_2(\bar{\mathcal{B}},x_z) \in N_\epsilon(\alpha)$ whenever $x_z \in (0,\delta)$. Thus, $MP_2(\mathcal{B},x_z) - MP_2(\bar{\mathcal{B}},x_z) >$ whenever $x_z \in (0,\delta)$. Hence, technological change is increasingly clean for all $x_z \in (0,\delta)$. ■

Proof. Lemma 8: Suppose $\Psi(\mathcal{A},\mathcal{B},\mathbf{x}_o,z) \notin [m^{min}(\mathcal{A},\mathcal{B},\mathbf{x}_o,\mathbf{x}_z),\ M^{max}(\mathcal{A},\mathcal{B},\mathbf{x}_o,\mathbf{x}_z)]$. Then two cases arise

1. $\Psi(\mathcal{A},\mathcal{B},\mathbf{x}_o,z) < m^{min}(\mathcal{A},\mathcal{B},\mathbf{x}_o,\mathbf{x}_z)$

Given the definition of $m^{min}(\mathcal{A},\mathcal{B},\mathbf{x}_o,\mathbf{x}_z)$ and the fact that the sectoral MAC is the common MAC under allocative efficiency, the following is true for all $u=1,\ldots,U$:

$$\Psi(\mathcal{A},\mathcal{B},\mathbf{x}_o,z)=MAC(\mathcal{A},\mathcal{B},x_o^u,\mathfrak{X}_z^u)=\frac{MP_1(\mathcal{A},x_o^u,\mathfrak{X}_z^u)}{MP_2(\mathcal{B},\mathfrak{X}_z^u)}<\frac{MP_1(\mathcal{A},x_o^u,x_z^u)}{MP_2(\mathcal{B},x_z^u)}=MAC(\mathcal{A},\mathcal{B},x_o^u,x_z^u),$$

where $\mathfrak{X}_z^u=\mathfrak{X}_z^u\left(\mathcal{A},\mathcal{B},x_o^1,\ldots,x_o^U,z\right)$ for all $u=1,\ldots,U$. The above in conjunction with Remark 7 implies that $\mathfrak{X}_z^u>x_z^u$ for all $u=1,\ldots,U$. But given that g is increasing in x_z, this implies that

$$z=\sum_{u=1}^U\mathfrak{Z}^u=\sum_{u=1}^U g(\mathcal{B},\mathfrak{X}_z^u)\ >\ \sum_{u=1}^U g(\mathcal{B},x_z^u)=\sum_{u=1}^U z^u=z,$$

where $\mathfrak{Z}^u=\mathfrak{Z}^u\left(\mathcal{A},\mathcal{B},x_o^1,\ldots,x_o^U,z\right)$ for all $u=1,\ldots,U$ is as defined in (9). A contradiction is clearly seen. Hence, this case is ruled out.

2. $\Psi(\mathcal{A},\mathcal{B},\mathbf{x}_o,z)>M^{max}(\mathcal{A},\mathcal{B},\mathbf{x}_o,\mathbf{x}_z)$

   Given the definition of $M^{max}(\mathcal{A},\mathcal{B},\mathbf{x}_o,\mathbf{x}_z)$ and the fact that the sectoral MAC is the common MAC under allocative efficiency, the following is true for all $u=1,\ldots,U$:

$$\Psi(\mathcal{A},\mathcal{B},\mathbf{x}_o,z)=MAC(\mathcal{A},\mathcal{B},x_o^u,\mathfrak{X}_z^u)=\frac{MP_1(\mathcal{A},x_o^u,\mathfrak{X}_z^u)}{MP_2(\mathcal{B},\mathfrak{X}_z^u)}>\frac{MP_1(\mathcal{A},x_o^u,x_z^u)}{MP_2(\mathcal{B},x_z^u)}=MAC(\mathcal{A},\mathcal{B},x_o^u,x_z^u).$$

The above in conjunction with Remark 7 implies that $\mathfrak{X}_z^u<x_z^u$ for all $u=1,\ldots,U$. But given that g is increasing in x_z, this implies that

$$z=\sum_{u=1}^U\mathfrak{Z}^u=\sum_{u=1}^U g(\mathcal{B},\mathfrak{X}_z^u)\ <\ \sum_{u=1}^U g(\mathcal{B},x_z^u)=\sum_{u=1}^U z^u=z,$$

where $\mathfrak{Z}^u=\mathfrak{Z}^u\left(\mathcal{A},\mathcal{B},x_o^1,\ldots,x_o^U,z\right)$ is as defined in (9) for all $u=1,\ldots,U$. A contradiction arises. Hence, this case is also ruled out.

■

Proof. Theorem 17:

Step 1.

We first prove that $\gamma=\bar{\gamma}^{min}\left(\hat{t},t,\mathbf{x}_o,z\right)\in\mathfrak{MAC}\left(\hat{t},t,\mathbf{x}_o,z\right)$: Suppose $\gamma\notin\mathfrak{MAC}\left(\hat{t},t,\mathbf{x}_o,z\right)$. By the definition of γ there exists $u\in\{1,\ldots,U\}$ such that $\gamma=\bar{\gamma}\left(\hat{t},t,x_o^u,\mathfrak{X}_z^u\right)$, where $\mathfrak{X}_z^u=\mathfrak{X}_z^u\left(\hat{t},t,\mathbf{x}_o,z\right)$ for all plants $u=1,\ldots,U$. Since $\gamma\notin\mathfrak{MAC}\left(\hat{t},t,\mathbf{x}_o,z\right)$, there exists $\hat{u}$ such that $\gamma\notin\left[\ \underline{\gamma}\left(\hat{t},t,x_o^{\hat{u}},\mathfrak{X}_z^{\hat{u}}\right),\ \bar{\gamma}\left(\hat{t},t,x_o^{\hat{u}},\mathfrak{X}_z^{\hat{u}}\right)\ \right]=:\left[\underline{\gamma}^{\hat{u}},\bar{\gamma}^{\hat{u}}\right]$. Together with the fact that $\gamma=\min\{\bar{\gamma}\left(\hat{t},t,x_o^1,x_z^1\right),\ \ldots,\ \bar{\gamma}\left(\hat{t},t,x_o^U,\mathfrak{X}_z^U\right)\}$ this implies that $\underline{\gamma}^u\leq\bar{\gamma}^u=\gamma<\underline{\gamma}^{\hat{u}}\leq\bar{\gamma}^{\hat{u}}$, where $\underline{\gamma}^u=\underline{\gamma}\left(\hat{t},t,x_o^u,\mathfrak{X}_z^u\right)$ and $\bar{\gamma}^u=\bar{\gamma}\left(\hat{t},t,x_o^u,\mathfrak{X}_z^u\right)$. This implies $\left[\underline{\gamma}^u,\bar{\gamma}^u\right]\cap\left[\underline{\gamma}^{\hat{u}},\bar{\gamma}^{\hat{u}}\right]=\emptyset$, contradicting the fact that $\mathfrak{MAC}\left(\hat{t},t,\mathbf{x}_o,z\right)\neq\emptyset$. Hence, $\gamma\in\mathfrak{MAC}\left(\hat{t},t,\mathbf{x}_o,z\right)$.

Step 2.

We now prove that $\gamma = \underline{\gamma}^{max}\left(\hat{t}, t, \mathbf{x}_o, z\right) \in \mathfrak{MAC}\left(\hat{t}, t, \mathbf{x}_o, z\right)$: Suppose $\gamma \notin \mathfrak{MAC}\left(\hat{t}, t, \mathbf{x}_o, z\right)$. By the definition of γ there exists $u \in \{1, \ldots, U\}$ such that $\gamma = \underline{\gamma}\left(\hat{t}, t, x_o^u, \mathfrak{X}_z^u\right)$. Since $\gamma \notin \mathfrak{MAC}\left(\hat{t}, t, \mathbf{x}_o, z\right)$, there exists $\hat{u}$ such that $\gamma \notin \left[\ \underline{\gamma}\left(\hat{t}, t, x_o^{\hat{u}}, \mathfrak{X}_z^{\hat{u}}\right),\ \bar{\gamma}\left(\hat{t}, t, x_o^{\hat{u}}, \mathfrak{X}_z^{\hat{u}}\right)\ \right] =: \left[\underline{\gamma}^{\hat{u}}, \bar{\gamma}^{\hat{u}}\right]$. Together with the fact that $\gamma = \max\{\underline{\gamma}\left(\hat{t}, t, x_o^1, x_z^1\right),\ \ldots,\ \underline{\gamma}\left(\hat{t}, t, x_o^U, \mathfrak{X}_z^U\right)\}$ this implies that $\underline{\gamma}^{\hat{u}} \leq \bar{\gamma}^{\hat{u}} < \gamma = \underline{\gamma}^u \leq \bar{\gamma}^u$, where $\underline{\gamma}^u = \underline{\gamma}\left(\hat{t}, t, x_o^u, \mathfrak{X}_z^u\right)$ and $\bar{\gamma}^u = \bar{\gamma}\left(\hat{t}, t, x_o^u, \mathfrak{X}_z^u\right)$. This implies $\left[\underline{\gamma}^u, \bar{\gamma}^u\right] \cap \left[\underline{\gamma}^{\hat{u}}, \bar{\gamma}^{\hat{u}}\right] = \emptyset$, contradicting the fact that $\mathfrak{MAC}\left(\hat{t}, t, \mathbf{x}_o, z\right) \neq \emptyset$. Hence, $\gamma \in \mathfrak{MAC}\left(\hat{t}, t, \mathbf{x}_o, z\right)$.

Step 3.

Next we prove that $\underline{\gamma}^{max} := \underline{\gamma}^{max}\left(\hat{t}, t, \mathbf{x}_o, z\right) \leq \bar{\gamma}^{min}\left(\hat{t}, t, \mathbf{x}_o, z\right) =: \bar{\gamma}^{min}$. Suppose to the contrary $\underline{\gamma}^{max} > \bar{\gamma}^{min}$. There exist $\underline{u}$ and $\bar{u}$ such that $\underline{\gamma}^{max} = \underline{\gamma}(\hat{t}, t, x_o^{\underline{u}}, \mathfrak{X}_z^{\underline{u}}) =: \underline{\gamma}^{\underline{u}}$ and $\bar{\gamma}^{min} = \bar{\gamma}(\hat{t}, t, x_o^{\bar{u}}, \mathfrak{X}_z^{\bar{u}}) =: \bar{\gamma}^{\bar{u}}$. Then $\underline{\gamma}^{\bar{u}} \leq \bar{\gamma}^{\bar{u}} = \bar{\gamma}^{min} < \underline{\gamma}^{max} = \underline{\gamma}^{\underline{u}} \leq \bar{\gamma}^{\underline{u}}$, where $\bar{\gamma}(\hat{t}, t, x_o^{\underline{u}}, \mathfrak{X}_z^{\underline{u}}) =: \bar{\gamma}^{\underline{u}}$ and $\underline{\gamma}^{\bar{u}}(\hat{t}, t, x_o^{\bar{u}}, \mathfrak{X}_z^{\bar{u}}) =: \underline{\gamma}^{\bar{u}}$. But that means $[\underline{\gamma}^{\bar{u}}, \bar{\gamma}^{\bar{u}}] \cap [\underline{\gamma}^{\underline{u}}, \bar{\gamma}^{\underline{u}}] = \emptyset$, which is a contradiction to $\mathfrak{MAC}\left(\hat{t}, t, \mathbf{x}_o, z\right) \equiv\neq \emptyset$ (see definition of $\mathfrak{MAC}\left(\hat{t}, t, \mathbf{x}_o, z\right)$ in (28) and Remark 16).

Step 4.

We now show that $\left[\underline{\gamma}^{max},\ \bar{\gamma}^{min}\right] \subseteq \mathfrak{MAC}\left(\hat{t}, t, \mathbf{x}_o, z\right)$. From Steps 1 and 2 we have that both $\underline{\gamma}^{max}$ and $\bar{\gamma}^{min}$ are in $\mathfrak{MAC}\left(\hat{t}, t, \mathbf{x}_o, z\right)$. As an intersection of closed intervals in $\mathbf{R}$, $\mathfrak{MAC}\left(\hat{t}, t, \mathbf{x}_o, z\right)$ is a convex set. Hence, $\left[\underline{\gamma}^{max},\ \bar{\gamma}^{min}\right] \subseteq \mathfrak{MAC}\left(\hat{t}, t, \mathbf{x}_o, z\right)$.

Step 5.

We now show that $\left[\underline{\gamma}^{max},\ \bar{\gamma}^{min}\right] = \mathfrak{MAC}\left(\hat{t}, t, \mathbf{x}_o, z\right)$. Suppose to the contrary that there exists $\gamma \in \mathfrak{MAC}\left(\hat{t}, t, \mathbf{x}_o, z\right)$ such that $\gamma \notin \left[\underline{\gamma}^{max},\ \bar{\gamma}^{min}\right]$. Two cases arise

- $\gamma > \bar{\gamma}^{min}$. There exists $\bar{u}$ such that $\bar{\gamma}^{min} = \bar{\gamma}(\hat{t}, t, x_o^{\bar{u}}, \mathfrak{X}_z^{\bar{u}}) =: \bar{\gamma}^{\bar{u}}$. Let $\underline{\gamma}^{\bar{u}}(\hat{t}, t, x_o^{\bar{u}}, \mathfrak{X}_z^{\bar{u}}) =: \underline{\gamma}^{\bar{u}}$. Then we have $\underline{\gamma}^{\bar{u}} \leq \bar{\gamma}^{\bar{u}} = \bar{\gamma}^{min} < \gamma$. Thus, $\gamma \notin \left[\underline{\gamma}^{\bar{u}}, \bar{\gamma}^{\bar{u}}\right]$, which is a contradiction to $\gamma \in \mathfrak{MAC}\left(\hat{t}, t, \mathbf{x}_o, z\right) = \bigcap_{u=1}^{U}\left[\ \underline{\gamma}\left(\hat{t}, t, x_o^u, \mathfrak{X}_z^u\right),\ \bar{\gamma}\left(\hat{t}, t, x_o^u, \mathfrak{X}_z^u\right)\ \right]$.

- Suppose $\gamma < \underline{\gamma}^{max}$. There exists $\underline{u}$ such that $\underline{\gamma}^{max} = \underline{\gamma}(\hat{t}, t, x_o^{\underline{u}}, \mathfrak{X}_z^{\underline{u}}) =: \underline{\gamma}^{\underline{u}}$. Let $\bar{\gamma}(\hat{t}, t, x_o^{\underline{u}}, \mathfrak{X}_z^{\underline{u}}) =: \bar{\gamma}^{\underline{u}}$. Then we have $\gamma < \underline{\gamma}^{max} = \underline{\gamma}^{\underline{u}} \leq \bar{\gamma}^{\underline{u}}$. Hence, we have $\gamma \notin \left[\underline{\gamma}^{\underline{u}}, \bar{\gamma}^{\underline{u}}\right]$, which is a contradiction to $\gamma \in \mathfrak{MAC}\left(\hat{t}, t, \mathbf{x}_o, z\right) = \bigcap_{u=1}^{U}\left[\ \underline{\gamma}\left(\hat{t}, t, x_o^u, \mathfrak{X}_z^u\right),\ \bar{\gamma}\left(\hat{t}, t, x_o^u, \mathfrak{X}_z^u\right)\ \right]$.

■

Table A 1: Upper and lower bounds of plant-level MAC values under technical efficiency

	2010		2015	
Plant ID	$\underline{\gamma}^u$	$\bar{\gamma}^u$	$\underline{\gamma}^u$	$\bar{\gamma}^u$
1	847.5	847.5	0	753.5
3	1260.7	1260.7		
4	0	149.6	1046.6	1046.6
5	0	149.6	1179.0	1179.0
6	1260.7	1260.7	0	629.3
7	1260.7	1260.7	1046.6	1046.6
8	977.7	977.7	1046.6	1046.6
9			1046.6	1046.6
10	977.7	977.7	1046.6	1046.6
11	0	787.7	0	708.9
12	977.7	977.7	752.6	752.6
13	1260.7	1260.7	1285.0	1285.0
14	977.7	977.7	1046.6	1046.6
15	0	149.6	1179.0	1179.0
16	987.1	1260.7	1046.6	1179.0
17	0	149.6	1179.0	1179.0
18	0	787.7	1285.0	1285.0
19	977.7	977.7	1045.2	1045.2
20			0	1285.0
21	0	149.6	1046.6	1046.6
22	977.7	977.7	1179.0	1179.0
23	0	467.2	629.3	629.3
24	0	149.6	0	629.3
25	861.4	861.4	0	629.3
26	0	117.1	752.6	752.6
28	0	149.6	1046.6	1046.6
29	0	149.6	0	708.9
30	0	149.6	1046.6	1046.6
31	0	117.1	1046.6	1046.6
33	0	149.6	1046.6	1046.6
34	0	117.1	752.6	752.6
35	0	912.8		
36	458.7	458.7	0	629.3
37	0	149.6	1046.6	1046.6
38	0	117.1	917.8	919.0
39	0	117.1	1046.6	1046.6
40	802.3	802.3	1046.6	1046.6
41	0	117.1	1046.6	1046.6
42	0	912.8	1285.0	1285.0
43	0	117.1	752.6	752.6
44	0	458.7	752.6	752.6
45	0	117.1	728.7	728.7
46	0	117.1	752.6	752.6
47	0	117.1	0	629.3
48	1248.7	1248.7	1046.6	1046.6
49	0	117.1	629.3	629.3
50	0	914.2	0	917.8
51	0	117.1	1046.6	1046.6

Table A 2: Upper and lower bounds of plant-level MAC values under allocative efficiency

	2010		2015	
Plant ID	$\underline{\gamma}^u$	$\bar{\gamma}^u$	$\underline{\gamma}^u$	$\bar{\gamma}^u$
1	847.5	977.7	1046.6	1046.6
3	787.7	1260.7		
4	787.7	1260.7	752.6	1046.6
5	787.7	1260.7	847.8	1179.0
6	787.7	1260.7	752.6	1046.6
7	585.8	1024.7	752.6	1046.6
8	861.4	977.7	752.6	1046.6
9			752.6	1046.6
10	861.4	977.7	752.6	1046.6
11	787.7	1260.7	847.8	1179.0
12	861.4	977.7	752.6	1046.6
13	787.7	1260.7	847.8	1179.0
14	861.4	977.7	752.6	1046.6
15	787.7	1260.7	847.8	1179.0
16	616.7	987.1	752.6	1046.6
17	787.7	1260.7	847.8	1179.0
18	787.7	1260.7	847.8	1179.0
19	861.4	977.7	1045.2	1046.6
20			0.0	1285.0
21	787.7	1260.7	752.6	1046.6
22	861.4	977.7	752.6	1046.6
23	859.9	977.7	752.6	1046.6
24	787.7	1260.7	752.6	1046.6
25	861.4	977.7	752.6	1046.6
26	861.4	977.7	752.6	1046.6
28	787.7	1260.7	752.6	1046.6
29	787.7	1260.7	847.8	1179.0
30	787.7	1260.7	752.6	1046.6
31	861.4	977.7	919.0	1046.6
33	787.7	1260.7	752.6	1046.6
34	861.4	977.7	752.6	1046.6
35	912.8	1460.8		
36	861.4	977.7	1045.2	1046.6
37	787.7	1260.7	752.6	1046.6
38	861.4	977.7	919.0	1046.6
39	861.4	977.7	752.6	1046.6
40	861.4	977.7	752.6	1046.6
41	861.4	977.7	752.6	1046.6
42	912.8	1460.8	0.0	1285.0
43	117.1	987.1	752.6	1046.6
44	861.4	977.7	752.6	1046.6
45	861.4	977.7	1045.2	1046.6
46	861.4	977.7	752.6	1046.6
47	861.4	977.7	752.6	1046.6
48	585.8	1024.7	752.6	1046.6
49	861.4	977.7	752.6	1046.6
50	914.2	914.2	919.0	1046.6
51	861.4	977.7	752.6	1046.6

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